Math 1500 Review for midterm 4

Topics:

- Series: geometric series; convergence tests, power series
- Fourier series
- Complex numbers
- Differential equations: linear ODE's, homogeneous and inhomogeneous

Extra questions:

1. Does the following series converge or diverge?

$$\sum \frac{\sin(n)}{n^2} \qquad \sum \ln\left(1+\frac{1}{n}\right) \qquad \sum \frac{\ln n}{n^2} \qquad \sum (-1)^n \frac{\ln n}{n}$$
$$\sum \frac{3^n}{3^n+4^n} \qquad \sum \sin(1/n) \qquad \sum \cos(1/n) - 1 \qquad \sum \frac{n!}{(n+2)!+1}$$

2. For what values of x do the following series converge absolutely? converge conditionally? diverge?

$$\sum \frac{(2x-3)^n}{n} \qquad \sum \frac{(2x-3)^n}{n(n+1)}$$

3. Write the (infinite) Taylor series of the following functions around $x = 0$.

$$\frac{1}{\sqrt{1+x}}; \qquad \frac{1}{\sqrt{1-x^2}}.$$

In each case, state the radius of convergence of the resulting series. What about the endpoints?

4. Evaluate the following sums explicitly:

$$\sum_{n=1}^{\infty} n2^{-n}; \qquad \sum_{n=1}^{\infty} \frac{1}{(n+2)\,3^n}.$$

5. Suppose that $f(x)$ is an odd function on the interval $[-L, L]$ that may be respresented as

$$f(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{x}{L}\pi n\right).$$

Determine a_n in terms of $f(x)$. Then evaluate a_n explicitly if $f(x) = x$.

6. Determine the real and imaginary part of $\left(1+i\sqrt{3}\right)^{2010}$. Hint: use polar coordinates.

7. A sequence x_n is defined recursively by

$$x_0 = 0, \quad x_1 = 1; \qquad (1)$$
$$x_n = 2x_{n-1} + x_{n-2}, \quad n \geq 2. \qquad (2)$$

   (a) Write down x_0 to x_6.
   (b) Find all solutions to (2) of the form $x_n = \lambda^n$ for some λ.
   (c) Using part (b), show that $x_n = C_1\lambda_1^n + C_2\lambda_2^n$ for some constants C_1, C_2 and for λ_1, λ_2 as found in part (b). Determine the constants C_1, C_2.

(d) Determine x_{2010}.

8. Find the solution to the following initial value problemssa:

$$y'' - 2y' + 5y = 0, \quad y(0) = 1,\ y'(0) = 1.$$

$$y'' - 2y' + y = 0, \quad y(1) = 1, y'(1) = 0.$$

$$y''' - y = 0, \quad y(0) = 0 = y'(0), \quad y''(0) = -1.$$

9. Find a particular solution for each of the ODE below.

$$\begin{aligned} y'' - 2y' + 5y &= \exp(x) \\ y'' - 2y' + 5y &= \exp((1+2i)x) \\ y'' - 2y' + 5y &= \exp(x)\sin 2x \end{aligned}$$