Math 2001 Final Exam, December 2012.

1. Find the equation of the tangent line to the curve $\vec{r}(t) = (2\cos t, 3\sin t, -t)$ at $t = \pi$.

2. Let L_1 be the line given in parametric form by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} t + \begin{pmatrix} 2012 \\ 2013 \\ 2001 \end{pmatrix}$. Let L_2 be the line that goes through the points $(0,0,1)$ and $(1,0,1)$. Determine the equation of the plane which is parallel to the lines L_1 and L_2 and which goes through the point $(1,0,0)$.

3. Show that
$$\lim_{(x,y)\to(0,0)} \frac{y^2 + \sin x^2}{x^2 - y^2}$$
does not exist. **BONUS**: change the "minus" in the above formula into a "plus". What happens then?

4. Consider the parametric curve $r = 1 + \cos(2\theta)$, $0 \le \theta \le 2\pi$.
   (a) Sketch it.
   (b) Find the area enclosed by this curve.
   (c) Find the equation of the tangent line to this curve at $\theta = \pi/4$.

5. Consider the space curve $\vec{r}(t) = \left(t, \frac{t^2}{2}, t^2\right)$.
   (a) Find the unit normal N and the unit tangent T for this curve at $t = 0$.
   (b) Find the curvature κ at $t = 0$ for this curve.
   (c) Let $\vec{a} = \left.\frac{d^2\vec{r}}{dt^2}\right|_{t=0}$ be the acceleration at $t = 0$. It can be written as $\vec{a} = a_T T + a_N N$ where T, N are as found in part (a). Determine the constants a_T and a_N.
   (d) Find the equation of the tangent plane to the surface given by $z = x^2 - \ln(yx)$ at $(x,y) = (1,1)$.
   (e) Using part (a), approximate the value of z when $x = 1.1$ and $y = 1.2$.

6. Let $f(x,y) = x^2 - y^2 - 2x + 4y - 3$.
   (a) Determine the critical points of $f(x,y)$.
   (b) Classify any critical points as either a local max, a local min, or a saddle point.

7. Determine the point on the ellipse $\left(\frac{x}{2}\right)^2 + y^2 = 1$ which is closest to the point $(2,1)$

8. Find $\int\int_D xy\,dA$ where D is the region bounded by the x-axis, y-axis and the line $x + y = 1$.

9. Let D be the region inside the circle $x^2 + y^2 = 4$, outside the circle $x^2 + y^2 = 1$, and with $x, y \ge 0$. A lamina has the shape D and a non-uniform density given by $\rho(x,y) = \sqrt{x^2 + y^2}$. Determine its mass and its center of mass.