Homework 2

1. (a) Sketch the the curves $r = \sin\theta$ and $r = \sqrt{3}\cos\theta$.

   (b) Find the area of the region that lies inside the two curves.

2. (a) Find an equation of the sphere that passes through the point $(-2, 3, 0)$ and has center $(2, -1, 2)$.

   (b) Describe the curve in which this sphere intersects the plane $y = 1$.

3. Find two unit vectors that are orthogonal to both $\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

4. (a) Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6, 1)$.

   (b) Find the unit vectors that are perpendicular to the tangent line.

   (c) Sketch the curve $y = 2\sin x$ and the vectors in parts (a) and (b), all starting at $(\pi/6, 1)$.

5. (a) Find two unit vectors that make an angle of $60°$ with $\mathbf{v} = \langle 3, 4\rangle$.

   (b) Find the acute angle between the curves $y = \sin x$ and $y = \cos x$ at their point of intersection for $0 \le x \le \pi/2$. (The angle between two curves is the angle between their tangent lines at the point of interection.)

6. Show that if $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal, then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.

7. Use the scalar triple product to determine whether the points $A = (1, 3, 2)$, $B = (3, -1, 6)$, $C = (5, 2, 0)$, and $D = (3, 6, -4)$ lie in the same plane.

8. Given the points $A = (1, 0, 1)$, $B = (2, 3, 0)$, $C = (-1, 1, 4)$, and $D = (0, 3, 2)$, find the volume of the parallelepiped with adjacent edges AB, AC, and AD.