Homework 3

1. Consider a triangle whose vertices are $P = (1, 0, 0)$, $Q = (0, 1, 0)$ and $R = (0, 0, 2)$.

   (a) Sketch the triangle PQR in three dimensional space.

   (b) Find a vector that is in the direction perpendicular to the plane through PQR.

   (c) Find the area of triangle PQR.

   (d) Find the equation of the line that is perpendicular to the plane through PQR and goes through the point $(1, 1, 1)$.

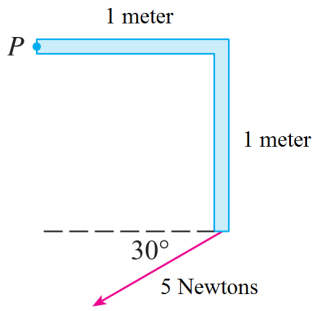
2. Let P be a point not on the plane that passes through the points Q, R, S.

   (a) Show that the distance d from to the point P to the plane through Q, R, S is

   $$d = \frac{|a \cdot (b \times c)|}{|a \times b|}$$

   where $a = QR$, $b = QS$, and $c = QP$.

3. Find the magnitude of the torque about P if a 5-N force is applied as shown. Also find the direction of the torque vector.



4. The points $P = (0, 0, 0)$, $Q = (1, -1, 1)$, $R = (1, 0, 0)$ and $S = (1, 2, a)$ all lie on the same plane. Determine the value of a. Hint: use the formula for the volume of a parallelepiped.

5. (a) Find the equation of the line which is parallel to the planes $x + y + z = 1$ and $x = 2$, and which goes through the origin.

   (b) Find the equation of the line which is the intersection of the planes in part (a).

6. Determine whether the lines

   $$L_1 : x = \frac{y-1}{-1} = \frac{z-2}{3}, \qquad L_2 : \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$$

   are parallel, intersecting, or skew. If they intersect, find the point of intersection.

7. (a) Find equations for the line of intersection of the planes $3x - 2y + z - 1 = 0$, $2x + y - 3z - 3 = 0$. (b) Find the acute angle between these planes.

8. For each of the conic below, indentify its type and sketch it. If it's an ellipse/circle make sure to indicate its radii and center. For hyperbolas, indicate their center (and directions).

   (a) $x^2 + 4x + y^2 = 0$

   (b) $4x^2 - 8x + y^2 - 4y = 1$

   (c) $4x^2 - 8x - y^2 + 4y = 4$

   (d) $-4x^2 + 8x + y^2 - 4y = 4$

9. Sketch the graphs of the following quadratic surfaces. For each of these, classify them (e.g. one-sheet hyperboloid, two-sheet hyperboloid, paraboloid, ellisposoid, saddle...).

   (a) $x^2 - y^2 + z^2 = 1$

   (b) $x^2 - y^2 + z^2 = -1$

   (c) $x^2 - y + z^2 = 1$

   (d) $x^2 - 2x - \frac{y^2}{4} + z^2 = 0$