Homework 4

1. (a) Find a vector function that represents the curve of intersection of the paraboloid $z = 4x^2 + y^2$ and the cylinder $y = x^2$.

   (b) Find the derivative of the curve you found in (a) and the unit tangent vector at $t = 1$.

2. Find the point on the curve $r(t) = \langle 2\cos t, 2\sin t, e^t \rangle$, $0 \le t \le \pi$, where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

3. Evaluate the integrals:

   (a) $\int (t\,\mathbf{i} + \frac{1}{t+1}\,\mathbf{j} - t\ln t\,\mathbf{k})\,dt$

   (b) $\int_0^{\pi/4} (\sec^2 t\,\mathbf{i} + \cos^2 t\,\mathbf{j} + 2t\,\mathbf{k})\,dt$

4. (a) Find the unit tangent and the unit normal vectors $T(t)$ and $N(t)$ of the curve $r(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t \rangle$ for $t > 0$.

   (b) Find the curvature of $r(t)$.

   (c) Find the curvature at the point $(\frac{\pi^2}{4}, 1, \frac{\pi}{2})$.

5. Find the curvature of $y = x^4$ at the point $(\frac{1}{2}, \frac{1}{16})$.

6. Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

7. Find the tangential and normal components of the acceleration vector of $r(t) = \frac{1}{t}\,\mathbf{i} + \frac{1}{t^2}\,\mathbf{j} + \frac{1}{t^3}\,\mathbf{k}$ at the point $(1, 1, 1)$.

8. Draw a contour map of the function $f(x, y) = \ln(x^2 + 4y^2)$.