## Homework 5

1. (a) Find the tangent plane approximation to the function $f(x,y) = x^3y - y^2 + 5$ at the point $(x,y) = (1,1)$. (b) Approximate $f(1.2, 0.9)$ using linear approximation you found in part (a).

2. 3Suppose that $u(x,y) = F(x-2y) + G(x+2y)$.

   (a) Compute $u_x, u_y$

   (b) Compute $u_{xx}, u_{yy}$

   (c) Find the value of $k$ such that $u_{xx} = ku_{yy}$.

3. The wind-chill index is modeled by the function

$$W = 13 + 0.6T - 11v^{0.15} + 0.4Tv^{0.15}$$

   where $T$ is the temperature and $v$ is the wind speed. When $T = -15^0$C and $v = 30$ km/h, by how much would you expect the apparent temperature to drop if (a) the actual temperature decreases by $2^0$C? (b) the wind speed increases by 2 km/h?

4. You are given a function with the property that $f_x(x,y) = 2x + ayx^2$ and $f_y = x^3 + 1$. Determine the constant $a$. Hint: use equality of mixed partial derivatives.

5. Given a surface defined implicitly through

$$x^2 - y^2 + z^2 + xy^3z = 2.$$

   (a) Determine $\partial z/\partial x$ at the point $(x,y,z) = (1,1,1)$.

   (b) Determine $\partial z/\partial y$ at the point $(x,y,z) = (1,1,1)$.

   (c) Determine the equation of the plane tanget to this surface at the point $(x,y,z) = (1,1,1)$.

6. The wave heights $h$ in the open sea depend on the speed $v$ of the wind and the length of time $t$ that the wind has been blowing at that speed. Values of the function $h = h(v,t)$ are recorded in feet in the following table.

Duration (hours)

| Wind speed (knots) $v$ \ $t$ | 5 | 10 | 15 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|---|
| 10 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 15 | 4 | 4 | 5 | 5 | 5 | 5 | 5 |
| 20 | 5 | 7 | 8 | 8 | 9 | 9 | 9 |
| 30 | 9 | 13 | 16 | 17 | 18 | 19 | 19 |
| 40 | 14 | 21 | 25 | 28 | 31 | 33 | 33 |
| 50 | 19 | 29 | 36 | 40 | 45 | 48 | 50 |
| 60 | 24 | 37 | 47 | 54 | 62 | 67 | 69 |

(a) On the $t - v$ plane, make a rough sketch of several well-chosen contours of $h(v, t)$. Indicate the values of contours chosen. Hint: you can print out the table above and draw the contours directly on the table.

(b) Estimate the values of $h_v(30, 20)$ and $h_t(30, 20)$. What is the practical interpretation of these values?

(c) Using part (c) and linear approximation, estimate $h(33, 25)$ as best as you can.

7. Let $u(x, t)$ be the temperature of a metal bar at a one-dimensional location $x$ and time $t$. Then $u(x, t)$ satisfies the *heat equation* which has the following non-dimensional form:

$$u_t = u_{xx}$$

(a) There is a solution of the Heat equation of form $u(x, t) = e^{-\lambda t} \sin(mx)$. Determine a relationship between $\lambda$ and $m$.

(b) Find *all* solutions of the heat equation that have the form $u(x, t) = F(t) \sin(mx)$ for some function $F(t)$. Show that they are all some scalar multiple of the solution you found in part (a).

(c) Can you find any other solutions of the heat equation that are *not* of the form $u(x, t) = F(t) \sin(mx)$?

8. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.3T$, where $P$ is measured in kilopascals, $V$ in liters, and $T$ in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 10 L to 12 L and the temperature decreases from 310 K to 305 K.