Homework 6

1. (a) Use chain rule to find the partial derivatives $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ of $z = e^{x^2 y}$, where $x(u,v) = \sqrt{uv}$ and $y(u,v) = \frac{1}{v}$.

   (b) Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ only in terms of u and v and evaluate at $(u,v) = (1,1)$.

2. (a) Find the directional derivative of $f(x,y,z) = xy^2z^3$ at $P(2,1,1)$ in the direction from P to $Q(0,-3,5)$.

   (b) In which direction is the directional derivative maximized? What is the steepest rate of increase of f at P?

3. Consider the surface $xy + yz + zx = 5$. Find (a) the tangent plane at $(1,2,1)$ and (b) the normal line at $(1,2,1)$.

4. Find all critical points of the function

$$f(x,y) = x^3 + y^2 - 2xy + x - 2y.$$

   Use the second derivative test to classify the critical points as either min, max or a saddle point. If it is a min or max, is it a global min or max?

5. Find dimensions of the box without a lid with volume 32cm^3 that has minimal surface area.

6. Find the extreme values of $f(x,y) = 2x^2 + 3y^2 - 4x - 5$ on the region $x^2 + y^2 \leq 16$.

7. The total production of a certain product is modeled by the Cobb-Douglas function $P = 100L^{3/4}K^{1/4}$, where L represents the units of labor and K represents the units of capital. Each labor unit costs \$200 and each capital unit costs \$250. If the total expenses for labor and capital cannot exceed \$50,000, find the maximum level of production.

8. Use Lagrange multipliers to prove that the triangle with maximum area that has a given perimeter p is equilateral.
   Hint: Use Heron's formula for the area

$$A = \sqrt{s(s-x)(s-y)(s-z)}$$

   where $s = \frac{p}{2}$ and x, y, z are the lengths of the sides.

9. The plane $2x + 2y + z = 2$ intersects the surface $z = x^2 + y^2$. Use Lagrange multipliers to:

   (a) Find the point of intersection of these two surfaces which is closest to the z-axis.

   (b) Find the point of intersection which is furthest away from the z-axis.