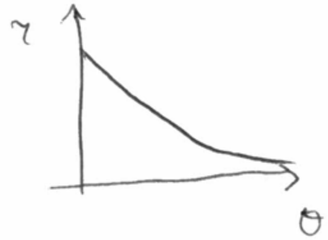
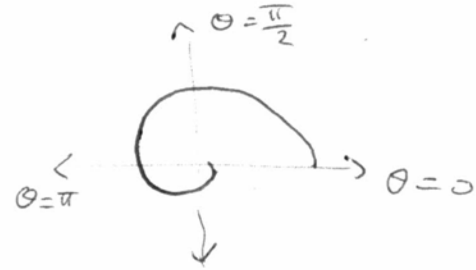
1)



$\longrightarrow$



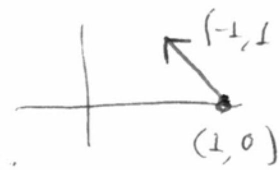
$r = e^{-\theta}$; $\quad x = r\cos\theta$, $\quad y = r\sin\theta$

$x' = r'c - rs = e^{-\theta}(-\cos\theta - \sin\theta)$

$y' = r's + rc = e^{-\theta}(-\sin\theta + \cos\theta)$

<u>At $\theta = 0$</u> : $r = 1$, $x = 1$, $y = 0$

and $x' = -1$, $y' = 1$



So equation of tangent line is:

$\cancel{y = (x-1)}$

$y = -x + 1$.

<u>Vertical tangent</u> $\iff$ $x' = 0 \iff \cos\theta = -\sin\theta$

<u>or</u> $\tan\theta = -1$

$\iff \theta = \frac{\pi}{2} + \frac{\pi}{4}$ or $\theta = -\frac{\pi}{4} + 2\pi$.

2) It has form $n \cdot (p - p_0) = 0$ where $p_0 = (0,0,0)$, $p = (x,y,z)$

and $n = (\hat{i} + \hat{k}) \times (\hat{j} + \hat{k})$

$= \underbrace{\hat{i}\times\hat{j}}_{\hat{k}} + \underbrace{\hat{i}\times\hat{k}}_{-\hat{j}} + \underbrace{\hat{k}\times\hat{j}}_{-\hat{i}} + \underbrace{\hat{k}\times\hat{k}}_{0}$

$= (-1, -1, 1)$

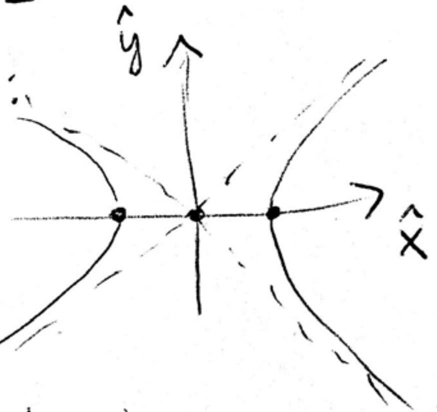
$\Rightarrow \boxed{-x - y + z = 0}$

3 a) Complete square: $y^2-4y=(y-2)^2-4$

So that
$$\begin{aligned}&4x^2-y^2+4y-5\\ =\;&4x^2-(y^2-4y)-5\\ =\;&4x^2-(y-2)^2-1=0\end{aligned}$$

So let $\hat{x}=2x$, $\hat{y}=y-2$, then

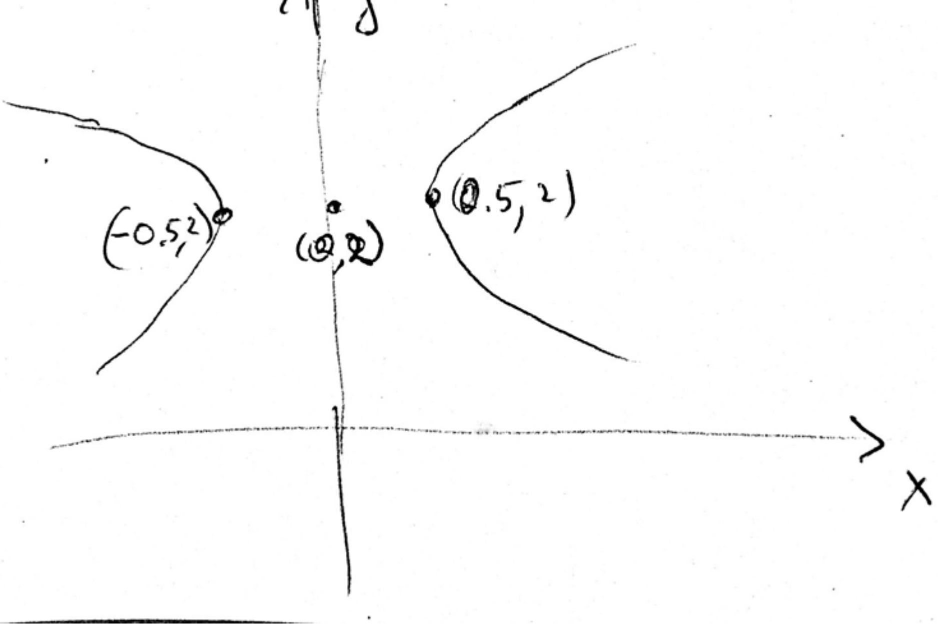
$\hat{x}^2-\hat{y}^2=1$ which is hyperbola like this:



The center $\hat{x}=0, \hat{y}=0$ correspons to $x=0$, $y=2$.

The vertices $\hat{x}=\pm1, \hat{y}=0$ corresponds to $x=\pm\frac{1}{2}, y=2$.

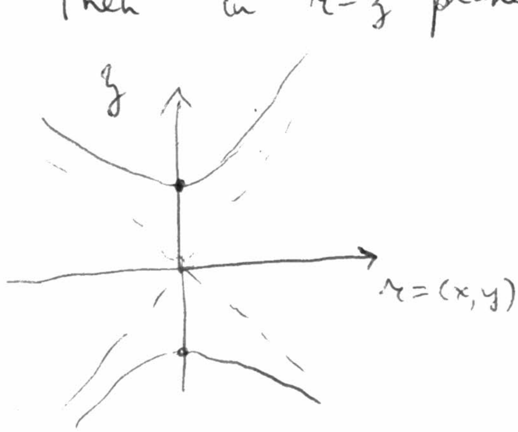
So in original variables, the hyperbola is:



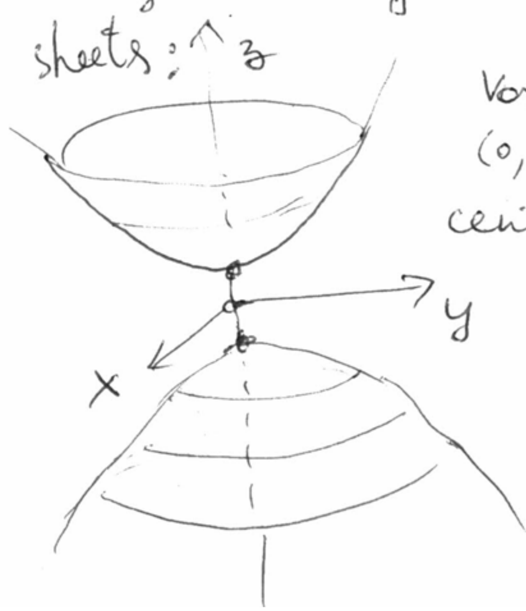
(b) Let $r^2 = x^2 + y^2$. Then in $r-z$ plane, we have:

$r^2 - z^2 = -1$

is a hyperbola:



So rotating it around the z-axis we get a hyperbola of two sheets:



Vortices at $(0,0,\pm 1)$, centered at $(0,0,0)$.

4)

$r' = (1, 2t, 2)$

$r'' = (0, 2, 0)$

$\kappa = \dfrac{r' \times r''}{|r'|^3}$; $\left. r' \times r'' = \right|_{t=0} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix} = -4\hat{i} - \hat{j} + \hat{k}$

$|r' \times r''| = \sqrt{16+1+1} = \sqrt{18}$

$|r'| = \sqrt{1+4} = \sqrt{5}$

$$\boxed{\kappa = \frac{\sqrt{18}}{5^{\frac{3}{2}}}}$$

b) $T = \frac{(1, 2t, 2)}{|(1, 2t, 2)|} = \frac{(1, 2t, 2)}{\sqrt{5+4t^2}} \Rightarrow T|_{t=0} = \frac{(1, 0, 2)}{\sqrt{5}}$.

$T'|_{t=0} = (0, 2, 0) \Rightarrow N = \frac{T'}{|T'|} = (0, 1, 0)$.

$B = T \times N = \frac{1}{\sqrt{5}}(\hat{\imath} + 2\hat{k}) \times \hat{\jmath} = \frac{1}{\sqrt{5}}(\hat{k} - 2\hat{\imath})$.

c) $a = s'T + \kappa s^2 N$; $s = 1$, $s' = 0$

$\Rightarrow a = \kappa N = \frac{\sqrt{18}}{5^{\frac{3}{2}}} \hat{\jmath}$.