Sample practice questions for preparation of the final exam

- **Topics covered:**
  - Parametric curves/vector functions (chap 10, 13)
    * 2d, 3d, polar
    * Quadratic curves, surfaces: completing square, classification in 2d and 3d, sketching
    * areas, tangents, arclength
    * Curvature, tension, $\hat{T}, \hat{N}, \hat{B}$, acceleration
  - Vectors, lines, planes (chap 12):
    * dot, cross product
    * Computing equations of planes and lines through points etc
    * Projections
    * Distances (eg plane and line, line and line etc)
  - Partial derivatives (chap 14.1-6)
    * limits, continuity
    * Tangent planes and linear approximation
    * Chain rule
    * Directional derivatives
  - Optimization problems (chap 14.7-8)
    * 1st, 2nd derivative test
    * Classification (saddle/local max/min...)
    * Lagrange multipliers
  - Integration in 2d (chap 15)
    * Double integrals over rectangles and general regions
    * Polar coordinates.
- **How to study:** Go over midterms, homeworks, sample questions and old final exam
- **Additonal questions below.**

1. Find the area which of the region enclosed by the curve $x = t^2, y = \sin t$, $0 \leq t \leq 1$, the x-axis and the line $x = 1$.
2. Given the space curve $\vec{r}(t) = (\cos t, \sin t, t)$.
   (a) Find the unit normal, unit tangent and the unit bi-normal to this curve.
   (b) Find its curvature.
   (c) Re-parameterize it using the arclength.
   (d) Determine the equation of the line tangent to this curve at $t = 0$.

3. Consider the curve whose equation in polar coordinates is given by $r(\theta) = 1 + \theta$, $0 \le \theta \le \pi$.

   (a) Sketch this curve.

   (b) Find the area bounded by this curve and the x-axis.

   (c) Find the length of this curve.

4. Consider a plane going through points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 2)$. Determine the equation of a line which is perpendicular to this plane and which goes through the point $(1, 1, 1)$.

5. Find the arclength of the parametric curve $x = e^t \sin t$, $y = e^t \cos t$ with $0 \le t \le 1$.

6. A function $F(x, y)$ has the properties that $F(1, 1) = 1$, $F_x(1, 1) = 2$ and $F_y(1, 1) = 3$. Let $f(t) = F(t^2, t^3)$. Determine $f'(1)$.

7. Find the distance between the plane whose normal is $(1, 2, 3)$ and which goes through the point $(1, 0, 0)$, and the origin.

8. Show that
$$\lim_{(x,y)\to(0,0)} \frac{y + 2x^2}{x^2 + y^2}$$
   does not exist.

9. The box was measured and it was found that its dimensions were $1, 2$, and 3 meters. The error in measurements was 1% in each direction. What is the error in the calculated volume of this box?

10. (a) Find the equation of the tangent plane to the surface given by $z = \sqrt{yx}$ at $(x, y) = (1, 4)$.
    (b) Using part (a), approximate the value of z when $x = 1.1$ and $y = 3.9$.

11. The plane $x + y + 2z = 2$ intersects the hyperboloid $z = x^2 - y^2$. Find the point on the intersection of these two surfaces which are the closest to the origin.

12. Find $\int\int_D x dA$ where D is the domain bounded by the ellipse $y^2 + (x/2)^2 = 1$ with x and $y \ge 0$.

13. Find the mass and the center of mass for the rectangular plate $0 \le x \le 2, 0 \le y \le 1$, whose density is $\rho(x, y) = 1 + x + y$.

14. (a) Sketch the region D which is the larger of the two regions that you get when you slice a disk $x^2 + y^2 \le 2x$ by the line $y = x$.

    (b) Write the integral $\int_D f(x, y) dA$ as an iterated integral using ordering $dxdy$ and then using ordering $dydx$.

15. **More old-exam questions:**

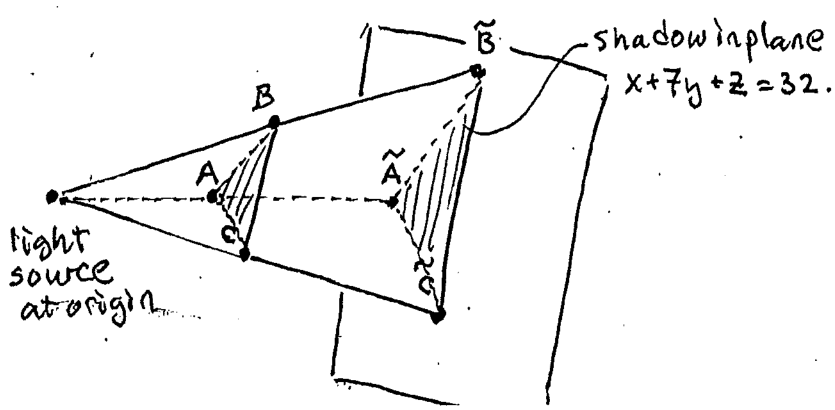
1. Let $A = (0,2,2), B = (2,2,2), C = (5,2,1)$

3pts (a) The line which contains A and is perpendicular to the triangle ABC has parametric equations:

$$\begin{cases} x = \\ y = \\ z = \end{cases}$$

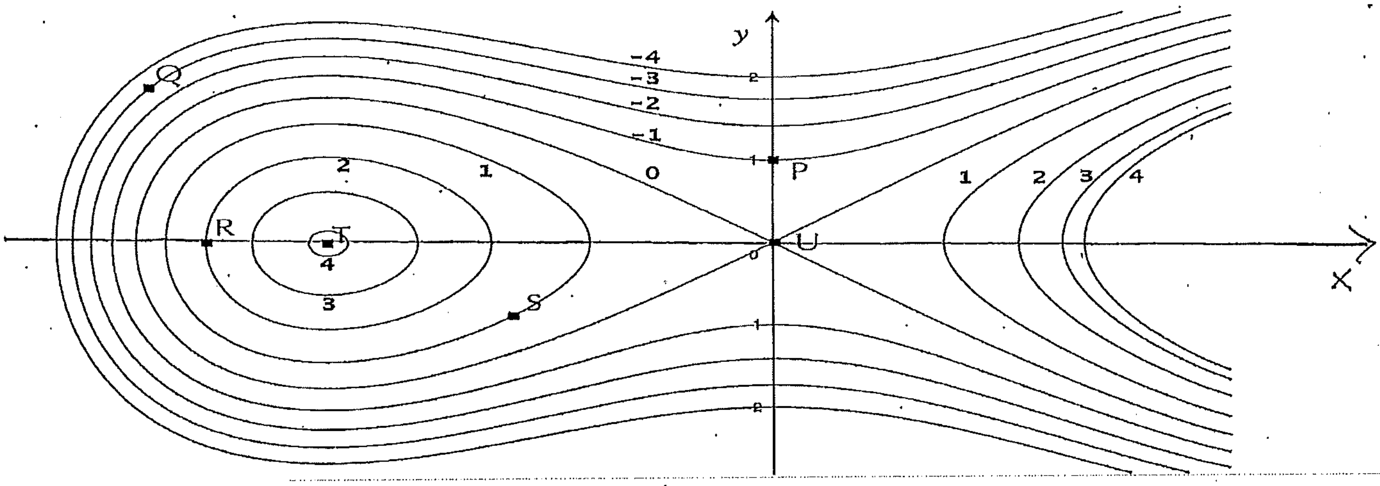
3pts (b) The set of all points P such that $\overrightarrow{PA}$ is perpendicular to $\overrightarrow{PB}$ form a Plane/ Line/ Sphere/ Cone/ Paraboloid/ Hyperboloid (circle one) in space which satisfies the equation: ____________________,

3pts (c) If a light source at the origin shines on triangle ABC making a shadow on the plane $x + 7y + z = 32$ (see diagram), then $\tilde{A} = (\_\_\_, \_\_\_, \_\_\_)$.



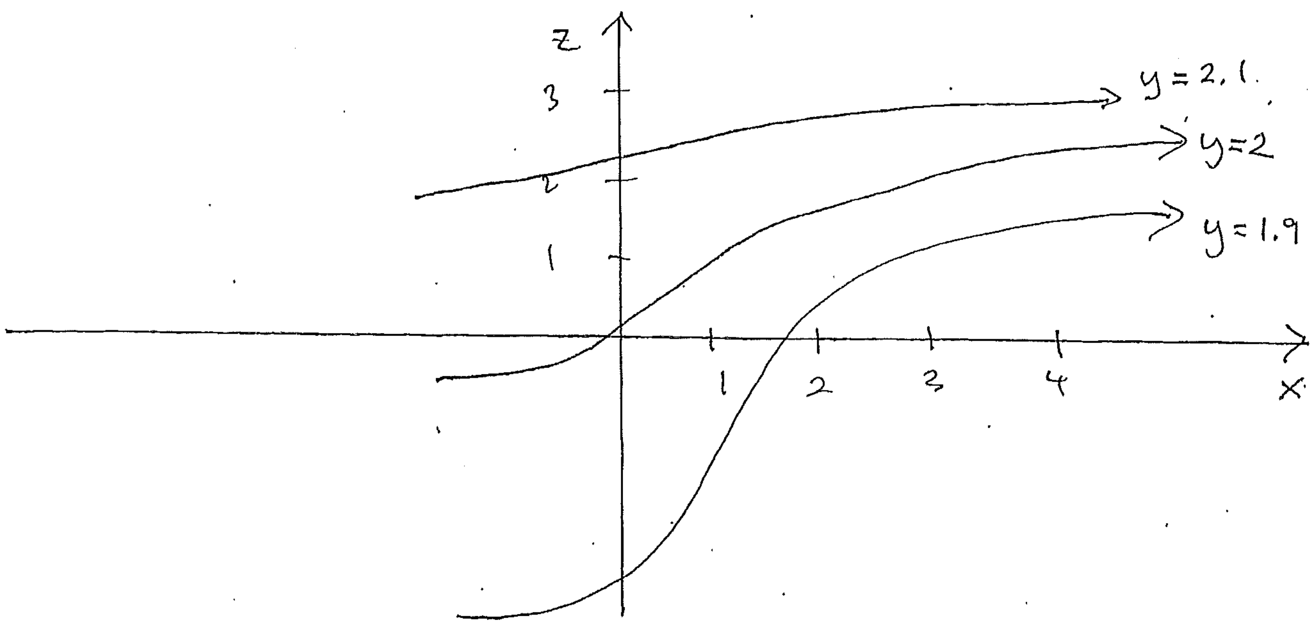
2a) Some level curves of a function f(x, y) are plotted in the xy plane below. For each of the four statements below, circle the letters of **all** points in the diagram where the situation applies. For example, if the statement were "These points are on the y-axis:" you would circle both P and U, but none of the other letters. You may assume that a local maximum occurs at Point T.

i) gradient f is zero	P R S T U
ii)f has a saddle point	P R S T U
iii) the partial derivative f_y is positive	Q R S T U
iv) the directional derivative of f in the direction <0,-1> is negative	Q R S T U



2b)The diagram below shows three ``y traces" of a graph z=F(x, y) plotted on xz axes. (namely, the intersections of the surface z=F(x, y) with the three planes (y=1.9, y=2, y=2.1). For each statement below, circle the correct word.

i) The first order partial derivative F_x (1,2) is	positive/negative/zero (circle one)
ii) F has a critical point at (2,2)	true/false (circle one)
iii) The second orderpartial derivative F_{xy} (1,2) is	positive/negative/zero (circle one)



3. Consider the functions $F(x, y, z) = z^3 + xy^2 + xz$ and $G(x, y, z) = 3x - y + 4z$. You are standing at the point $P(0, 1, 2)$.

5pts (a) If you jump from P to $Q(0.1, 0.9, 1.8)$ then the amount by which F changes is approximately:

______________________________ (use linear approximation).

3pts (b) If you jump from P in the direction along which G increases most rapidly, then F will increase/decrease (circle one and explain below).

3pts (c) You jump from P in a direction $\langle a, b, c\rangle$ along which rate of change of F and G are both zero. An example of such a direction is $\langle a, b, c\rangle =$ ________________ (need not be unit vector).

4. Suppose $f(x, y)$ is twice differentiable (with $f_{xy} = f_{yx}$), and $x = r\cos\theta$ and $y = r\sin\theta$.

7pts (a) Fill blanks below in terms of functions depending on r and/or θ, and partial derivatives of f with respect to x, y.

$f_\theta =$ ______________________________

$f_r =$ ______________________________

$f_{r\theta} =$ ________ $f_x +$ ________ $f_y +$ ________ $f_{xx} +$ ________ $f_{xy} +$ ________ f_{yy}

2pts (b) Let $g(x, y)$ be another function satisfying $g_x = f_y; g_y = -f_x$. Fill blanks below with constants or functions depending on r and/or θ

$f_r =$ ____________ g_θ

$f_\theta =$ ____________ g_r

5. The temperature in the plane is given by $T(x, y) = e^y(x^2 + y^2)$.

(a) 3pts (i) To find the warmest and coolest points on the **circle** $x^2 + y^2 = 100$ using Lagrange multipliers, we must solve the following system:

$$\begin{cases} \\ \\ \\ \end{cases}$$

4pts (ii) By solving the above system, we conclude the warmest point on the **circle** is: ________________, an the coolest point is: ________________

(b) Take the same termperature function as in part (a) ($T(x, y) = e^y(x^2 + y^2)$.)

1pts (i) To find the critical points of $T(x, y)$ we must solve the following system:

$$\begin{cases} \\ \\ \end{cases}$$

3pts (ii) By solving the above system we conclude the critical points are: _________________________.

2pts (c) The coolest point on the solid disc $x^2 + y^2 \leq 100$ is __________________.

6. Let $I = \int_0^{\overset{1}{\cancel{\sqrt{\pi}/2}}} \int_{x^2}^{1} x^3 \sin{(y^3)} dydx$

2pts (a) Sketch the corresponding region of integration in the xy plane (label your sketch sufficeintly so that one could conversely use it to determine the limits of double integration)

5pts (a) Evaluate I.