## Sample questions: midterm preparation

- Do the suggested problems from the book (see course outline)
- Go over all the homeworks.
- Topics covered: everything up to and including chap 13. Main topics are:
  - parametric curves; areas, surfaces of solids of revolutions....
  - polar coords, area, arclength...
  - conics: 2d and 3d.
  - dot product, cross product, lines, planes, intersections, distances...
  - Space curves: velocity, accelaration, curvature...
- Some additional questions below.
- 1. Find the area of the region enclose by one loop of the polar curve  $r = \sin 4\theta$ . Answer:  $\pi/16$ .
- 2. Consider the curve

$$x(t) = 3t - t^3, \qquad y(t) = 3t^2, \quad 0 \le t \le 1$$

- (a) Sketch this curve.
- (b) Find the area bounded by this curve, the x-axis and the line x = 2 (answer: 6/5)
- (c) Compute the surface area of the solid generated by rotating this curve around the x axis. (answer:  $\frac{48}{5}\pi$ )
- (d) Compute the surface area of the solid generated by generated by rotating this curve around the y axis. (answer:  $11\pi$ )
- 3. Find the length of the polar curve  $r = 5^{\theta}$ ,  $0 \le \theta \le 2\pi$ . Answer:  $(5^{2\pi} 1)\sqrt{1 + \frac{1}{(\ln 5)^2}}$
- 4. Find the volume of the parallelepiped with adjacent edges PQ, PR, PS where P = (-2, 1, 0), Q = (2, 3, 2), R = (1, 4, -1) and S = (3, 6, 1). (answer: 16)
- 5. Find an equation of the plane passing through the origin and the points (2, -4, 6) and (5, 1, 3). Answer: -9x + 12y + 11z = 0.
- 6. Consider the point A = (4, 1, 3) and the plane P given by the equation x + 2y 3z = 2. (a) Find the plane which passes through the point A and is parallel to the plane P.

(b) Find the distance between the two parallel planes from part (a). (c) Find the equation of the line L of intersection of the plane P with the xz-plane.

(d) Find the distance from the point B with the coordinates (0, 3, -4) to the line L.

Answers: (a) x + 2y - 3z = -3; (b)  $\frac{5}{\sqrt{14}}$ ; (c)  $L = P_0 + t\vec{v}$  where  $P_0 = (2, 0, 0)$  and  $\vec{v} = (-3, 0, -1)$ . (many other answers are of course possible).

- 7. Sketch the following surfaces, and classify each one (i.e. paraboloid/ellipsoid/hyperbola 1 or 2 sheet/cone etc): (a)  $x^2 + z^2 = y$ , (b)  $x^2 z^2 = y$ , (c)  $x^2 z^2 = y^2$ , (d)  $x^2 z^2 = y^2 + 1$ , (e)  $x^2 z^2 = y^2 1$ , (f)  $x^2 z^2 = -4y^2 + 9$ .
- 8. What kind of surface is  $x^2 + y^2 + 4z^2 16z + 12 = 0$ ? Sketch it.
- 9. What equation describes a hyperboloid of one sheet; of two sheets?
- 10. A car is entering the circular road whose radius is 30 meters [think the Rotary in Halifax]. Its current speed is 36km/hr but it exits at 42km/hr, after having gone one full circle around the rotary. Estimate the change in the car's speed, and compute its acceleration (i.e. its normal and tangential components) just before the car exits rotary. (answer:  $a_T = |v|' \approx 0.0318m/sec^2$ ;  $a_N \approx 4.537m/sec^2$ )
- 11. Compute the curvature of the the space curve  $(t, t^2, 3t)$  at the point t = 0. Compute N, T, B at that point.
- 12. Find the equation of parabola that has a curvature 4 at the origin.
- 13. It takes about 8.3 minutes for the light from the sun to reach the earth. Estimate the sun's mass. You can use the following facts: the gravity between two objects of masses  $m_1, m_2$  separated by a distance r is given by  $F = m_1 m_2 G/r^2$  where  $G = 6.67 \times 10^{-11} \frac{m^3}{kg \ sec^2}$ ; Earth makes one full revolution around the sun in 1 year and the speed of light is  $\approx 3 \times 10^8$  m/sec. You may assume that the earth is on a circular orbit around the sun.