Some additional review questions

See also midterm review sheet, go over all quizzes and midterm, and suggested problems. All quizzes/midterm/reviews will be posted to the course website, http://www.mathstat.dal.ca/~tkolokol/classes/2002

1. State the Divergence, Green and Stoke's theorems.

2. If $F = x^2 z\mathbf{i} + (y^2 z + 3y)\,\mathbf{j} + x^2\mathbf{k}$, find the flux of F across the part of the ellipsoid $x^2 + y^2 + 4z^2 = 16$, where $z \geq 0$, oriented with upward normal.

3. If $F(x, y, z) = (-z, x, y)$, what are the possible values of the line integral $\int_C F \cdot d\vec{x}$ around a circle of radius a in the plane $2x + y + 2z = 7$?

4. Find the maximum value of $\int_C F \cdot d\vec{x}$ where $F = xy^2\mathbf{i} + (3z - xy^2)\mathbf{j} + (4y - x^2 y)\mathbf{k}$ and C is a simple closed curve in the plane $x + y + z = 1$ oriented counterclockwise as seen from the high on the z-axis. What curve C gives this maximum?

5. Suppose that $F(x)$ is a smooth vector field $F : \mathbb{R}^3 \to \mathbb{R}$, which satisfies $|F(\vec{x})| \leq 1/(|\vec{x}|^3 + 1)$ for all $\vec{x} \in \mathbb{R}^3$. Show that
$$\int\int\int_{\mathbb{R}^3} \nabla \cdot F \; dV = 0.$$
Hint: integrate on a ball of radius R first, then take the limit $R \to \infty$.

6. Let $F = -y\mathbf{i} + x\cos(1 - x^2 - y^2)\,\mathbf{j} + yz\mathbf{k}$. Find the flux of *curl* F upwards through a surface whose boundary is the curve $x^2 + y^2 = 1, \;\; z = 2$.

7. Find the general solution to the following ODE's:
$$y'' - 4y' + 5y = 0$$
$$y'''' - 16y = 0$$

8. Find a particular solution to the following ODEs:
$$y'' + 4y = \sin t$$
$$y'' + 4y = \sin(2t)$$
$$y'' - 4y' + 5y = \cos(x)$$
$$y'' - 4y' + 5y = e^x + 2\cos(x)$$

9. Solve the following initial value problem:
$$y'' + 4y = \sin t, \quad y(0) = 2, \quad y'(0) = 0.$$

10. (a) The ODE $x^2 y'' + xy' - 4y = 0$ admits two independent solutions of the form $y(x) = x^p$ for some p. Find them.
    (b) Find the general solution to the ODE $x^2 y'' + xy' - 4y = 1/x^2$.

11. Use power series solutions to solve the initial value problem
$$y'' + xy' + y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$