

Math 2002 Midterm

1. Let E be the solid which is bounded by the cylinder $x^2 + y^2 = 1$, lies above the parabola $z = x^2 + y^2$ and below the plane $z = 2 + y$. Determine the volume of this solid.
2. Compute the surface integral $I = \iint_S z^2 dS$ where S is the surface of the unit sphere, $S : x^2 + y^2 + z^2 = 1$.
3. The simply connected region D has area 4 and its center of mass is located at $(0, 2)$. Evaluate the line integral

$$I = \int_C y^2 dx + (xy + x) dy$$

where C is the boundary of D , positively oriented.

4. The surface S is given in parametric form by

$$S : \begin{pmatrix} x(r, t) \\ y(r, t) \\ z(r, t) \end{pmatrix} = \begin{pmatrix} r \cos t \\ r \sin t \\ r \end{pmatrix}, \quad \text{with } 0 \leq r \leq 1 \text{ and } 0 \leq t \leq \pi.$$

- (a) Compute the unit normal to S .
- (b) Determine the surface area of S .
- (c) Determine the flux of the vector $F = (x, 0, 0)$ through S .