

# MATH 2300, Homework 1

- (a) Read Chapter Zero of the book (b) Install matlab on your laptop
- Estimate (a) how many school students (grade 0 to 12) are there in Nova Scotia? (b) The number of school teachers (c) The total education budget for schools of the province. Make sure to describe the methods and any assumptions you used to come up with your estimate. You can use internet but don't look up the exact number! (d) If you can, find the answers to (a,b,c) on the internet and compare to numbers you obtained. How does it stack up? What assumptions you think you missed?
- Consider the iteration  $x_0 = 1.0$ ,  $x_1 = \frac{1}{1+x_0}$ ,  $x_2 = \frac{1}{1+x_1}, \dots$ 
  - Using matlab, compute  $x_1, x_2, \dots, x_{20}$ . Hand in the printout.
  - Determine the limit of this iteration,  $x_\infty = \lim_{n \rightarrow \infty} x_n$ . [I'm looking for exact expression that you can derive by hand, not the numerical value you get from the computer!]
  - Using matlab, output the difference  $x_n - x_\infty$  for  $n = 1 \dots 20$ . Hand in the printout.
  - Based on the numerical results from part (c), make a guess about how close is  $x_{1000}$  to  $x_\infty$ ?
- Consider the following iteration:
$$x_{n+1} = 2x_n - ax_n^2$$
  - Set  $a = 2$ . Starting with  $x_0 = 0.1$ , use a computer to compute  $x_n$  for  $n = 1 \dots 8$ ; hand in your printout. Make any relevant observations. Please use `format long` to get the printout of more digits.
  - What is the limit of this iteration,  $x_\infty = \lim_{n \rightarrow \infty} x_n$  for any given  $a$ ? Note: assume that  $x_0$  is sufficiently close to  $x_\infty$ .
  - From part (b), propose a computer algorithm for division using only multiplication and addition/subtraction operations. Test your algorithm to compute  $\frac{1}{7}$ . How many operations were required to get 10 significant digits?
- You have a mortgage with a 5% interest rate and a student loan with 10% interest rate. Your mortgage debt is \$100,000 whereas your student debt is \$10,000. Your parents give you a gift of \$2000 to help you out with your debt. Should you pay it towards your mortgage or towards your student debt? Explain.
- You want to buy a house. Based on your salary, you can afford to pay a maximum of \$1000 per month for your mortgage. Assume the annual interest rate of 2.4%. What is the maximum price of the house you can afford if you wish to pay off the mortgage in 25 years? For simplicity, also assume that there is zero downpayment and no property taxes to pay.
- In a more realistic situation, assume that the downpayment is at least 10% of the house value (but you can put down more if you wish). Moreover, there are yearly municipal taxes which are 1.5% of the house purchase price per year. There are also "closing fees" (things like lawyer costs, moving expenses etc) estimated to be at \$6000. You have a maximum of \$40,000 in savings that you can use for a downpayment and closing costs. And you can afford a maximum of \$1000 of monthly payments towards the house (that is, mortgage+taxes). What is the maximum price of the house you can afford if you wish to pay off the mortgage in 25 years?
- You borrow \$200,000 from a bank. The interest rate is 5% per year. How large should your monthly payments be to pay off your loan in 25 years? Assume payments at the beginning of each month.
  - The prime interest rate is 4% (i.e. this is the Bank of Canada interest rate), so that the bank is making 1% interest on top of the prime interest rate (we will assume that the bank itself has no

money, so to give you the loan, it borrows \$200,000 from the central bank at a rate of 4%). How much profit will the bank make from your loan after the 25 years?

(c) Redo questions (a) and (b), but with the base interest rate of 1.5%, and the bank interest rate of 2.5%.

9. (a) Consider the dynamical system  $x_{n+1} = ae^{x_n}(1 - x_n) + x_n$  where  $a > 0$ . Convince yourself that  $x = 1$  is an equilibrium (fixed point) of this map. For which values of  $a$  is this fixed point stable and for which is it unstable?

(b) Use matlab to sketch the bifurcation diagram as  $a$  is increased. Hand in the printout.