MATH 2400, Homework 1

Due date: 20 September (Tuesday)

1. Install either Maple, Octave, matlab on your computer.

2. Using a computer program of your choice, plot the functions $y = x, y = x^2$ and $y = \sqrt{x}$ **on the same graph,** with $x \in [0, 2]$ and with $y \in [0, 2]$. Hand in the printout of your graph. Note: If using maple, the commands "display" and "plot" will be useful; if using matlab/octave, the commands "hold on" and "plot" will be useful. See help pages or google around.

3. Consider the iteration $x_0 = 1.0, \;\; x_1 = \frac{1}{1+x_0}, \;\; x_2 = \frac{1}{1+x_1}, \ldots$

   (a) Using a computer program, compute $x_1, x_2, \ldots x_{20}$. Hand in the printout.

   (b) Determine the limit of this iteration, $x_\infty = \lim_{n\to\infty} x_n$. [I'm looking for exact expression that you can derive by hand, not the numerical value you get from the computer! Hint: x_∞ satisfies $x_\infty = \frac{1}{1+x_\infty}$]

   (c) Using a computer, output the difference $x_n - x_\infty$ for $n = 1 \ldots 20$. Hand in the printout.

   (d) Based on the numerical results from part (c), make a guess about how close is x_{1000} to x_∞?

4. Ever wondered how computers perform division? One way is to use Newton's method. The idea is that the root of $f(x) = a - \frac{1}{x}$ is precisely $1/a$.

   (a) Apply Newton's method to $f(x) = a - \frac{1}{x}$ to come up with an algorithm that allows you to find $1/a$ using only multiplication and addition/substraction.

   (b) Illustrate your algorithm to compute $\frac{1}{7}$. Use $x_0 = 0.1$ as your starting point. How many iterations were required to get 8 digits? How many multiplication and addition operations? Make a printout of your iterations.

   (c) What happens if you use $x_0 = 1$ as your starting point for part (b)?

5. This question reviews the Taylor series.

   (a) Write down the first three non-zero terms of the Taylor series of $\ln(1 + h)$ centered at $h = 0$.

   (b) Using part (a), estimate $\ln(0.5)$ *without* using computers.

   (c) The Taylor Remainder theorem states that

   $$f(a+h) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!} h^n + \frac{f^{(N+1)}(\xi)}{(N+1)!} h^{N+1} \tag{1}$$

   where ξ is some number between a and $a + h$. Write out what this means for the case $a = 1$, $f(a + h) = \ln(1 + h)$; $N = 3$.

   (d) Using the Taylor Remainder theorem, estimate the error between the actual value of $\ln(0.5)$ and the value you got in part (b).