

MATH 2400 Midterm

No calculators or other aids. Write all answers in the booklet provided.

1. Set up Newton's method to determine $\sqrt[3]{2}$. Starting with $x_0 = 1$, compute x_1 .
2. The function $f(x)$ is tabulated below.

x	0	1	2
$f(x)$	1	1	0

- (a) Suppose that $f(x)$ is a quadratic. What is it?
 - (b) Suppose that $f(x)$ is some function such that $|f^{(3)}(x)| < 5$ for all x . Find some numbers a and b such that $a \leq f(0.5) \leq b$.
3. Let $f(x)$ be as tabulated in question 2.
 - (a) Estimate $\int_0^2 xf(x)dx$ using the Trapezoid rule.
 - (b) Suppose that it is known that $|f'(x)| \leq \left|x - \frac{2}{x^2+1}\right|$ and $|f''(x)| \leq \left|1 + \frac{4x}{(x^2+1)^2}\right|$. Estimate the error you made in part (a). Remark: if T_n is the Trapezoid rule with n subintervals, then $\left|T_n - \int_a^b f(x)dx\right| \leq \max |f''(x)| \frac{h^2(b-a)}{12}$.
 - (c) Use Romberg integration to estimate $\int_0^2 xf(x)dx$ as accurately as you can.
 4. Let $I(h) = \int_{-h}^h f(x)dx$ and let $N = f(-\frac{h}{2}) + f(\frac{h}{2})$, where f is a smooth function. Show that $|N - I| \leq Mh^p$ for some positive constants p and M . What is the value of the constants p and M ?