MATH 5250/4250, Homework 1

Due: Thursday, 18 September

1. Find a two-term expansion in ε of all roots of

$$x^2 + (2+\varepsilon)x + 1 + \varepsilon = 0$$

2. Find a two-term expansion in ε of all roots of

$$x^2 - 1 + \varepsilon \tanh(\frac{x}{\varepsilon})$$

3. Compute the first two terms of all four roots of

$$\varepsilon^2 x^4 - 2\varepsilon x^3 + x^2 - 2x + 1 = 0, \quad \varepsilon \ll 1.$$

4. Compute an asymptotic expansion of the root to

$$xe^{x^2} = \varepsilon^{-1}, \quad \varepsilon \ll 1$$

using an iteration method. Starting with the iteration $x_0 = \sqrt{\ln \varepsilon^{-1}}$, compute x_1 and x_2. Your result should be expressed in terms of $L_1 = \ln \varepsilon^{-1}$ and $L_2 = \ln L_1$. Compare your results with an exact numerical answer for $\varepsilon = 10^{-2}, 10^{-4}$ and 10^{-6}.

5. Find the first two terms of all large solutions to $\frac{1}{x} = \tan(x)$ in the limit $x \gg 1$. Hint: sketch the graph first.

6. Consider a curve given in polar coordinates by

$$r = 1 + \varepsilon \cos(N\theta), \quad \theta \in [0, 2\pi]$$

where N is relatively big and ε is small (sort of like a two-dimensional golf ball) Find a two-term expansion of the length of this curve if $\varepsilon \leq O(N^{-2})$. Why is this assumption necessary?