Midterm Sample questions

1. (a) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{(2-\sin\theta)^2}.$$

(b) Evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx.$$

2. (a) Determine the image of the unit disk $|z| < 1$ under the map

$$w = \frac{i+z}{i-z}.$$

(b) Determine the image of the unit disk $|z| < 1$ under the map

$$w = \frac{i}{2} \ln\left(\frac{i+z}{i-z}\right).$$

3. Solve $\Delta T = 0$ inside the region which consists of all of $\mathbb{C}$ minus two circles centered at ± 2 and having radius 1. Assume that $T = T_1$ on the boundary of the circle centered at -2 and $T = T_2$ on the boundary of the circle centered at $+2$. Sketch contour plot of your solution.

4. Solve $\Delta T = 0$ inside the the region D which consists of the positive half of the unit circle, $D = \{z : |z| < 1 \text{ and } \mathrm{Re}(z) > 0\}$. The boundary conditions are: $T = T_0$ for $z = e^{i\theta}, \theta \in (-\pi, \pi)$; $T = T_1$ for $z = iy, y \in (0,1)$; $T = T_2$ for $z = iy, y \in (-1,0)$; as shown on the blackboard.

- Do q.6 from HW5
- $\int_0^\infty \frac{\sin x}{x} dx = ??$
- $\int \frac{x^{\frac{1}{3}}}{x^2+4x+8} dx = ?$
- $\int_{-\infty}^{\infty} \frac{\cos x \, dx}{x^2+1} = ?$
- $\int_0^\infty \frac{\log x}{(1+x^2)^2} dx = ?$

Note: One of these questions will appear in the midterm.