MATH 2120, Homework 2

1. Consider a pond that initially contains 10 million gal of **fresh water**. Water containing an undesirable chemical flows into the pond at the rate of 5 million gal/yr, and the mixture in the pond flows out at the **same rate**. The concentration $\gamma(t)$ of chemical in the incoming water varies with time according to the expression $\gamma(t) = 2 + e^{-\frac{t}{2}}$ g/gal. Construct a mathematical model of this flow process and determine the amount of chemical in the pond at any time.

2. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of $200\,^\circ$ F when freshly poured, and 1 min later has cooled to $190\,^\circ$ F in a room at $70\,^\circ$ F, determine when the coffee reaches a temperature of $150\,^\circ$ F.

3. Find the general solution to $\left(\frac{y}{x} + 6x\right) + (\ln x + 2)y' = 0$ for $x > 0$.

4. Find the value b such that $(xy^2 + bx^2y) + (x+y)x^2y' = 0$ is exact, and then find the general solution to that exact equation.

5. Verify $r(x,y) = \dfrac{1}{xy^3}$ is an integrating factor for $x^2y^3 + x(1+y^2)y' = 0$ and find the general solution.

6. Use the integrating factor $r(x) = x$ to solve the differential equation $(3xy + y^2) + (x^2 + xy)y' = 0$.

7. Find an integrating factor $r(x)$ for $(4xy + 3y^2 - x) + x(x + 2y)y' = 0$.

8. The ODE $\left(3x^2y^3 + 2xy\right) + \left(2x^3y^2 + 3y^3\right)\frac{dy}{dx} = 0$ has an integrating factor of the form $\mu(x,y) = y^p$ for some constant p. Find this integrating factor and solve the ODE.