MATH 2120, Homework 4

1. (a) Write down a second order linear ODE that has the following particular solutions: $y_1 = e^x$, $y_2 = e^{-2x}$.

   (b) Write down a second order linear ODE with real coefficients that has the following particular solution: $y_1 = e^{-x}\sin(2x)$. What is another indepedent particular solution?

2. Write down a THIRD order linear ODE with real coefficients that has the following particular solutions: $y_1 = xe^x$; $y_2 = e^{-x}$ (why third order?). Then solve the resulting ODE subject to initial conditions $y(0) = 0, y'(0) = 0, y''(0) = 1$.

3. A small object of mass 1 kg is attached to a spring with spring constant 2 N/m. This spring-mass system is immersed in a viscous medium with damping constant 3 N$\cdot$ s/m. At time $t = 0$, the mass is lowered 1/2 m below its equilibrium position, and released. Find the position of the mass at any time t. Show that the mass will creep back to its equilibrium position without any oscillations as t increases.

4. A small object of mass 1 kg is attached to a spring with spring-constant 1 N/m and is immersed in a viscous medium with damping constant 2 N $\cdot$ s/m. At time $t = 0$, the mass is lowered 1/4 m and given an initial velocity of 1 m/s in the upward direction. Show that the mass will overshoot its equilibrium position once, and then creep back to equilibrium.

5. Using the mks units (meters-kilograms-seconds), suppose you have a spring with spring constant 4 N/m. You want to use it to weigh items. Assume no friction. You place the mass on the spring and put it in motion. a) You count and find that the frequency is 0.8 Hz (cycles per second). What is the mass? b) Find a formula for the mass m given the frequency ω in Hz.

6. (a) Find a particular solution to $y'' + y = 1$.

   (b) Find a particular solution to $y'' + y = x^2$.

   (c) Find a general solution to $y'' + y = 2 - 3x^2$.

   (d) Solve the initial value problem $y'' + y = 2 - 3x^2$, $y(0) = 0$, $y'(0) = 0$.

7. Find a particular solution to the ODE $y'' + y' = e^x \sin x$

8. Find a particular solution to $y'' - y = \exp(kx)$ where k is a real constant. NOTE: Your answer should have three distinct subcases: either $k = 1$, $k = -1$ or any other k [why?].

9. (a) Find a particular solution to $y'' + 9y = \cos(3x)$.

   (b) Solve the initial value problem $y'' + 9y = \cos(3x)$ with $y(0) = 0$ and $y'(0) = 1$.

10. (b) Find a particular solution to the ODE $y'' - y = x\exp(x)$

    (b) Solve the initial value problem $y'' - y = x\exp(x)$ with $y(0) = 0$ $y'(0) = 0$.