Mathematical model of crime hot-spots

- Criminal activity concentrates <u>non-uniformly</u>
  - e.g. "bad" neighborhood v.s. "good"
- Often, "hot-spots" of crime are observed
- Goal: model the formation of hot-spots mathematically.
  - can help police to use resources more efficiently
  - Prevent crime before it occurs?
- Paper by Short et.al., 2008 [1]:
  - Proposes a model that reproduces hot-spots
  - encorporates "<u>Repeat victimization</u>", "<u>Broken window</u>" theory of crime.

<u>Short et.al. model:</u>

Agent-Based model $\longrightarrow$ cellular automata model $\longrightarrow$ PDE model [continuum limit of C.A.]

- City is represented by a <u>lattice</u> $s = (i, j)$ 

  or $s = (i)$ 

- At each gridpoint, s, let

  $A_s(t) \equiv$ "attractiveness to crime" at time t

  $\rho_s(t) \equiv$ "number" (or density) of criminals

Modelling attractiveness to crime $A_s(t)$:

$$A_s(t) = \underbrace{A_0}_{\text{"background attractiveness"}} + \underbrace{B_s(t)}_{\text{"dynamic component" of attractiveness}}$$

- "Broken windows" theory: nearby crime affects current location.
  - Eg: If the house next door has a broken window, then this house is more likely to be broken into.

- Repeat offenses: criminals return to location that was previously victimized
  - Eg: "tagging" of Graffitti
- crime rate decays over time.

$$B_s(t+\delta t) = \Big[(1-\eta) B_s(t) + \underbrace{\frac{\eta}{2}(B_{s-1} + B_{s+1})}_{\text{"broken window"}}\Big]\underbrace{(1-\omega\,\delta t)}_{\text{decay}}$$

$$+ \underbrace{A_s \underbrace{(\delta t)\, \rho_s(t)}_{\text{expected crime events in time } [t,\, t+\delta t]}\, \theta}_{\text{"repeat offenses"}}$$

- $\eta \in (0,1)$ is the weight of "broken window" effect
- $\omega > 0$ is the decay rate
- $\theta > 0$ is the rate of offenses.

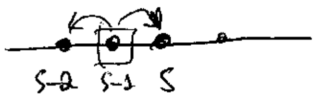
Modelling movement of criminals:

- Criminals move according to <u>biased random walk</u>:
  - They are more likely to move towards the area with higher attractiveness A_s
- At each time step, a criminal decides either to offend with probability p_s or else to move to a neighbouring site
- If the criminal commits a crime, they are removed from the site [goes home].
- Growth rate Γ.

$$\rho_s(t+\delta t) = \left(q_{(s-1)\to s}\,\rho_{s-1} + q_{(s+1)\to s}\,\rho_{s+1} \right)(1-p_s) + \Gamma(\delta t)$$

$q_{s'\to s} \equiv$ probability that a criminal at s' moves to site s

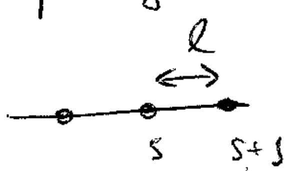
$$q_{(s-1)\to s} = \frac{A_s}{A_{s-2}+A_s} \; ; \quad q_{(s+1)\to s} = \frac{A_s}{A_{s+2}+A_s}$$



$p_s \equiv$ prob. that a criminal breaks in

$= (\delta t)\, A_s$

<u>Continuum limit:</u> Let ℓ = grid spacing and suppose $\ell \to 0$.



Estimate $\rho_s(t) \approx \rho(x,t)$ where $x = \ell s$;

$$\underset{B}{A_s}(t) \approx \underset{B}{A}(x,t).$$

Then
$$\frac{1}{2}\left(B_{s-1} + B_{s+1}\right) = \frac{1}{2}\left(B(x-\ell) + B(x+\ell)\right)$$
$$= B + \frac{\ell^2}{2} B_{xx} + O(\ell^3);$$
$$B(t+\delta t) = B + \delta t\, B_t + O((\delta t)^2);$$

$$\Rightarrow \quad B_t = \frac{\eta \ell^2}{2} B_{xx} - \omega B + \theta \rho A$$

$$\boxed{A_t = \frac{\eta \ell^2}{2} A_{xx} + \rho A \theta - \omega(A - A_0).}$$

To get continuum limit for ρ, we let

$$G(\ell) = \rho(x-\ell)\frac{A(x)}{A(x-2\ell)+A(x)} + \rho(x+\ell)\frac{A(x)}{A(x+2\ell)+A(x)}$$

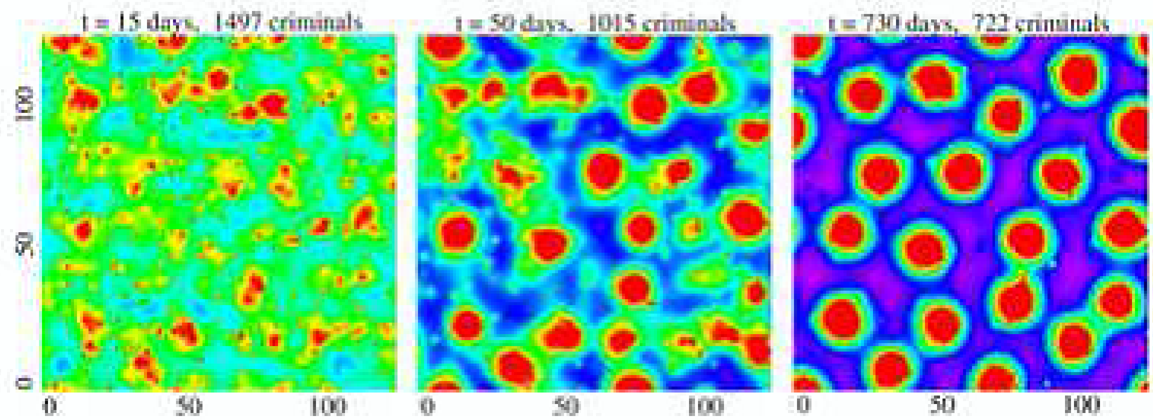
Then
$$G(0) = \rho(x);$$
$$G'(0) = 0;$$
$$G''(0) = \rho_{xx}(x) - 2\rho\frac{A_{xx}}{A} - 2\rho_x\frac{A_x}{A} + 2\rho\frac{A_x^2}{A^2}$$
$$= \rho_{xx} - \left(2\rho\frac{A_x}{A}\right)_x$$

$$\Rightarrow \quad \boxed{\rho_t = \frac{\ell^2}{2}\left(\rho_{xx} - \left(2\rho\frac{A_x}{A}\right)_x\right) - A\rho + \Gamma}$$

Data from LA police

Fig. 1. Dynamic changes in residential burglary hotspots for two consecutive three-month periods beginning June 2001 in Long Beach, CA. These density maps were created using ArcGIS.

Hot-spot formation using agent model



(Taken from [1])

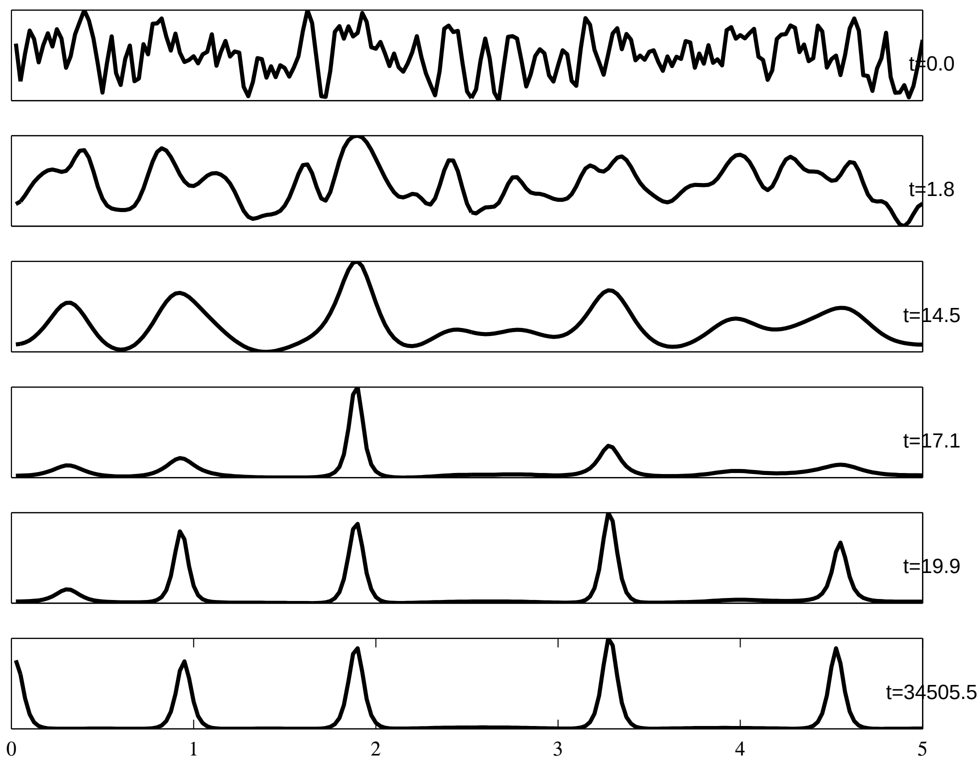
Dimensionless model:

$$A_t = \varepsilon^2 A_{xx} - A + \rho A + \alpha$$

$$\tau \rho_t = D\left(\rho_x - 2\frac{\rho}{A} A_x\right)_x - \rho A + \gamma - \alpha.$$

Example:

$\alpha = 1, \quad \gamma = 2, \quad D = 1, \ \varepsilon = 0.03.$



References:

[1] M. B. Short, M. R. D'Orsogna, V. B. Pasour, G. E. Tita, P. J. Brantingham, A. L. Bertozzi and L. B. Chayes (2008), A statistical model of criminal behavior, Math. Models. Meth. Appl. Sci., 18, Suppl. pp. 1249--1267.

[2] T. Kolokolnikov, M. Ward and J. Wei, The Stability of Steady-State Hot-Spot Patterns for a Reaction-Diffusion Model of Urban Crime., preprint.