1. Consider the Dirichlet Green's function on a square:

$$\Delta G = \delta\left(\vec{x} - \vec{x}_0\right), \quad \vec{x}, \vec{x}_0 \in D, \qquad G = 0 \text{ for } x \in \partial D \tag{1}$$

where $D = \{(x, y) : x \in (0, L)\,,\ y \in (0, H)\}$. Let R be the regular part, that is,

$$R\left(\vec{x}, \vec{x}_0\right) = G\left(\vec{x}, \vec{x}_0\right) - \frac{1}{2\pi} \ln\left|\vec{x} - \vec{x}_0\right|.$$

The goal is to use the method we discussed in class to determine $R_0 = R(c, c)$ where $c = (L/2, H/2)$ is at the center of the square.

(a) Decompose $\delta\left(\vec{x} - \vec{x}_0\right) = \sum v_m(x)\phi_m(y)$ and $G = \sum G_m(x)\phi_m(y)$ where $\phi_m(y)$ are the appropriate eigenfunctions (hint: $\phi_m(y) = \sin(\mathit{something})$).

(b) Find the solution to

$$u_{xx} - m^2 u = \delta(x - x_0), \quad u(0) = 0 = u(L)$$

(c) Using (a) and (b), solve for G_m

(d) Use the resummation technique we saw in class to obtain an infinite series expansion for R_0

(e) Test your result by using the formula obtained in (d), compute R_0 for $(L, H) = (2, 1)$ and $(L, H) = (1, 2)$. By symmetry, these should be the same. Take enough terms to get the answer to 3 significant digits. NOTE: the two Comment on convergence.

2. Consider the blow-up solution for

$$u_t = u_{xx} + e^u.$$

Suppose that blowup occurs first at $x = 0$ at some time $t = T$. Use the techniques from class to derive the blowup profile.