MATH 5230/4230, Homework 3

Due: Tues, Oct 26.

1. Consider the system
$$u_t + (2-u)u_x = 0; \qquad u(x,0) = 1 + \tanh(x).$$
   (a) Determine the characteristic curves for this ODE.
   (b) Show that the solution develops a shock and compute the time $t = t_s$ at which the shock first occurs.
   (c) Sketch the solution profile $u(x,t)$ for $t = 0,\ 0.5,\ 1,\ 1.5$.

2. Let
$$\phi(x) = \begin{cases} 1, & x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & x > 0. \end{cases} \qquad (1)$$
   (a) Consider the system
$$u_t + uu_x = 0; \quad u(x,0) = \phi(x).$$
   where ϕ is as in (1). The initial condition develops a shock at some time $t = t_s$. Compute the value of t_s and sketch the solution at $t = 0, t = t_s/2$ and $t = t_s$.
   (b) Repeat part (a) but with
$$u_t + uu_x = u; \quad u(x,0) = \phi(x).$$
   (c) Consider the system
$$u_t + uu_x - u = \varepsilon u_{xx}; \quad u(x,0) = \phi(x).$$
   with
$$\phi = \begin{cases} 1, x < 0 \\ 0, x > 0 \end{cases}.$$
   This system has a shock at $t = 0$, which is smoothed out by the diffusion term for $t > 0$. Compute the evolution equation for the location of the shock $x = s(t)$.
   (d) Repeat part (c) but with uu_x replaced by $u^2 u_x$.