

MATH 5230/4230, Homework 4

Due: Thurs, Nov 4

1. Consider the porous media PDE,

$$u_t = (uu_x)_x$$

Find a similarity solution which is even in x , with $\int_{-\infty}^{\infty} u(x, t) = M$ for all time t , and that has a compact support [i.e. $u > 0$ when $|x| < R$ and $u = 0$ if $|x| > R$, for some $R = R(t)$]. Sketch such a solution with $M = 1$ for several values of time t . Hint: try the ansatz $u(x, t) = t^a U(z)$ where $z = xt^b$ for a well chosen a, b .

2. Consider the blow-up solution for

$$u_t = u_{xx} + e^u.$$

- (a) Suppose that blowup occurs first at $x = 0$ at some time $t = T$. Apply the general formula derived in class to write down the blowup profile.
- (b) Make a change of variables as

$$u(x, t) = -\ln(T - t) + w(y, s)$$

where

$$y = \frac{x}{\sqrt{T-t}}; \quad s = -\ln(T-t).$$

Derive the equation for w ; then look for blow-up profile using linearization analysis. You should get the same answer as in question 2.