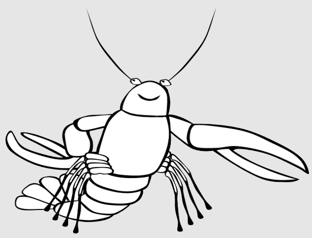
Workshop on Pattern Formation
Dalhousie University, Halifax, Canada
July 18-19, 2015
Followed by a week of research

Organizers:
Theodore Kolokolnikov (Dalhousie)
Ricardo Carretero (San Diego State University)
Michael Ward (University of British Columbia)

Workshop Schedule

All talks take place in Chase Building, Room 319.

	Saturday	Sunday
09:00 - 10:00	09:00 – 09:30 Panayotis Kevrekidis	09:00 – 09:30 Andrew J. Bernoff
	09:30 – 10:00 Ricardo Carretero	09:30 – 10:00 Alexandria Volkening
10:00 - 11:00	10:00 – 10:30 Roy Goodman	10:00 – 10:30 Shuangqian Xie
	10:30 – 11:00 **Coffee break**	10:30 – 11:00 **Coffee break**
11:00 - 12:00	11:00 – 11:30 Alanna Hoyer-Leitzel	11:00 – 11:30 Xiaofeng Ren
	11:30 – 12:00 Yana Nec	11:30 – 12:00 Matthias Winter
12:00 - 13:00	12:00 – 12:30 David Iron	12:00 – 12:30 Juncheng Wei
	12:30 – 14:00 **Lunch**	12:30 – 21:00 **Lunch research week starts**
13:00 - 14:00		
14:00 - 15:00	14:00 – 14:30 Alan Lindsay	
	14:30 – 15:00 Justin Tzou	
15:00 - 16:00	15:00 – 15:30 Daniel Gomez	
	15:30 – 16:00 **Coffee break**	
16:00 - 17:00	16:00 – 16:30 Mary Silber	
	16:30 – 17:00 Yuxin Chen	
17:00 - 18:00	17:00 – 21:00 **Lobster Boil Point Pleasant park follow the crowd**	
18:00 - 19:00		
19:00 - 20:00		
20:00 - 21:00		



Abstracts

Panayotis Kevrekidis

Existence, Stability and Dynamics of Solitary Waves and Vortices in Bose-Einstein Condensates: From Theory to Experiments

In this talk, we will present an overview of some of our recent theoretical, numerical and experimental efforts concerning the static, stability, bifurcation and dynamic properties of coherent structures that can emerge in one- and higher-dimensional settings within Bose-Einstein condensates. We will discuss how this ultracold setting can be approximated at a mean-field level by a deterministic PDE of the nonlinear Schrodinger type and what the fundamental nonlinear waves of the latter are, such as dark solitons and vortices. Then, we will try to go to a further layer of simplified description via nonlinear ODEs encompassing the dynamics of the waves within the traps that confine them, and the interactions between them. Finally, we will attempt to compare the analytical and numerical implementation of these reduced descriptions to recent experimental results and speculate towards a number of interesting possibilities for the future.

Ricardo Carreterro

Vortex pair configurations in Bose-Einstein condensates: theory and experiments.

We study the dynamics and stability of vortex configurations bearing a small number of vortices in harmonically trapped Bose-Einstein condensates. The dynamics is reduced to a system of quasi-particle ODEs describing their positions. Periodic and quasi-periodic solutions, and their stability, are studied and compared favorably with experimental observations. A symmetry-breaking bifurcation for regular polygonal states is identified and matched to experimental observations.

Roy Goodman

Hamiltonian Phenomena in NLS-Like Systems: Some Results, Some Plans, Some Ideas

The nonlinear-Schrodinger equation admits a Hamiltonian formulation, as do many finite-dimensional reduced systems based upon it. We show some examples where methods of Hamiltonian reduction allow us to discover and explain features in the dynamics. In a system of coupled optical waveguides, we show how relative periodic orbits and chaotic dynamics are affected by Hamiltonian Hopf bifurcations. We then discuss how techniques of Hamiltonian reduction and symmetry can be used to study vortex interactions in two-dimensional Bose-Einstein condensates. Time permitting, we will discuss some other ideas of additional phenomena we plan to explore and the methods we intend to use.

Alanna Hoyer-Leitzel

Using algebraic geometry to find bifurcations and symmetry in the n-vortex problem

Relative equilibrium solutions of the n-vortex problem are periodic solutions where the vortices maintain a fixed configuration as they rotate around the center of vorticity (analogous to center of mass in a gravitation problem). Under the assumption of one large and n small vortices, and specifically in the case when there are 3 small vortices, computational algebraic geometry can be used to examine the set of relative equilibrium solutions that would be hard to describe otherwise. After writing the equations for equilibrium solutions as polynomial equations, we can find bifurcations in the number of relative equilibria as the relative strengths of the small vortices change, and prove that symmetry in the configurations occurs only when the relative strengths of two of the small vortices are the same.

Yana Nec

Pattern formation with fractional derivatives

The talk will introduce fractional derivatives and explain how the regular calculus and normal diffusion are a special limit in a universe of anomalies. The peculiarities of different types of fractional derivatives and their suitability for particular analytical purposes will be discussed, alongside the classic notions of calculus that fall into abeyance.

Application of fractional derivatives to the generation of spike patterns with sub-diffusion and Lévy flights will delineated, with emphasis on the benefit to the parameter regime of pattern existence. Both types of anomaly will be compared to the regular diffusion as far as the shape, evolution and stability of the spike are concerned. The eigenvalue loci and their correspondence to enhanced or diminished stability of the system will be used to create a bigger picture regarding the pattern's behaviour.

No prior knowledge on fractional operators is required. Familiarity with PDEs and dynamical systems is of import.

David Iron

A model of receptor protein aggregation

We consider a model of the aggregation of cell receptor molecule aggregation in the cell membrane. We consider two cases. First we allow receptor molecules to diffuse freely on the cell membrane. Secondly we will consider density dependent diffusion. In this case large clusters of receptors will diffuse much more slowly than sparsely distributed ones.

Alan Lindsay

Vibrational Patterns of thin plates with clamped patches.

In this talk I will discuss the problem for the modes of vibration of a thin elastic plate with a collection of N small clamped patches. This talk will center on several fourth order eigenvalues problem and analysis of these in the limit of small patch size. These N patches represent defects in the plate and the main goal is to understand the effect of the number and location of these holes on the vibrational modes of the plate. The deviation of the eigenvalues from the patch free case are quantified and certain configurations which maximize this deviation for certain N are identified.

Justin Tzou

First passage times with mobile traps in one and two dimensions

Various problems in nature may be formulated in terms of mean first passage times (MFPT) of Brownian particles in the presence of traps. A typical example in cellular biology involves transport of molecules between the nucleus and cytoplasm of a cell. While most existing works focus on stationary traps, many scenarios may involve traps or targets that are non-stationary (e.g., predator-prey dynamics, search and rescue, diffusion-limited reactions). We discuss here the formulation of one and two-dimensional MFPT problems in the presence of mobile traps. In simple geometries, we exploit symmetries that allow for analysis by means of asymptotic methods. We also highlight some interesting and counter-intuitive results that arise due to the motion of the traps. Joint works with T. Kolokolnikov, A. Lindsay and S. Xie.

Daniel Gomez

Narrow Escape Problems in Non-Spherical Three-Dimensional Domains

The narrow escape problem (NEP) involves the calculation of the expected time, known as the mean first passage time (MFPT), for a randomly moving particle to escape a domain that is everywhere reflecting except at finitely many small holes through which the particle may escape. The typical approach to these problems involves developing asymptotic expansions for the MFPT and its spatial average that increases in accuracy as the holes become smaller. This talk addresses the NEP in three-dimensions for domains bounded by a level surface of an orthogonal coordinate system. Such domains include for example prolate and oblate spheroids, spheres, and biconcave disks. By using the method of matched asymptotic expansions and the singular behaviour of the surface Neumann Green's function in three-dimensional domains we obtain a two-term asymptotic expression for the average MFPT. The result depends on an assumption that remains an open problem, but is nevertheless supported by comparisons to numerical results.

Mary Silber

Pattern Formation in the Drylands: Self Organization in Semi-Arid Ecosystems

Much of our understanding of spontaneous pattern formation in spatially extended systems was developed in the wetlands" of fluid mechanics. That setting allowed well-controlled table-top laboratory experiments; it came with fundamental equations governing the system; it benefitted from a back-and-forth between theory and experiment. These investigations identified robust mechanisms for spontaneous pattern formation, and inspired the development of equivariant bifurcation theory. Recently, these pattern formation perspectives have been applied to modeling the vegetation in dryland ecosystems, where satellite images have revealed strikingly regular spatial patterns on large scales. Ecologists have proposed that characteristics of vegetation pattern formation in these water-limited ecosystems may serve as an early warning sign of impending desertification. We use the framework of equivariant bifurcation theory to investigate the mathematical robustness of this approach to probing an ecosystems robustness. Additionally, we identify new applied pattern formation research directions in this far-from-pristine setting, where there are no fundamental equations and no controlled laboratory experiments.

Yuxin Chen

Effects of the rate of precipitation changes on vegetation patterns

When faced with decrease in precipitation, vegetation in arid and semi-arid environments are prone to sudden irreversible changes, such as desertification, as the precipitation drops below a tipping point of the system. A possible coping mechanism is the formation of spatial patterns, which allows for concentration of sparse resources and the survival of the species within "ecological niches even below the tipping point of the homogeneous vegetation state. However, if the change in precipitation occurs too sudden, the system may not have time to transition to the patterned state and will pass through the tipping point, leading to extinction. We propose that the deciding factors are the rate of precipitation change and the amount of seasonal rainfall variability. We illustrate this phenomenon on a modified simple vegetation model proposed by Klausmeier.

Andrew J. Bernoff

Energy Driven Pattern Formation in Nearly Planar Fluid Systems

Nearly planar fluid systems include Langmuir layers, which are molecularly-thin polymer layers on a substrate (generally a quiescent subfluid) and Hele-Shaw systems where a thin fluid layer is confined between two glass plates. These systems are driven by intermolecular forces and damped by viscous dissipation. Their dynamics can often be described as dissipative gradient flows where the solution is driven toward energy minimizers. We describe two such systems. In the first, line tension (the two-dimensional analogue of surface tension) drives the fluid domains to become circular and the rate of relaxation to these circular domains can be used to deduce the magnitude of the line tension forces. The second system models the formation of convoluted fingered domains observed experimentally in ferrofluids for which pattern formation is driven by line tension and dipole-dipole repulsion. We asymptotically obtain an energy minimization problem depending only on a generalized line tension, Λ. Numerical studies yield a few highly symmetric stable shapes, but nothing that resembles the experimentally observed diversity. Adding a weak random background stabilizes a menagerie of domain morphologies recovering the diversity observed experimentally.

Alexandria Volkening

Modeling the formation of stripes in zebrafish

Zebrafish is a small fish with distinctive black and yellow stripes that form due to the interaction of different pigment cells. Working closely with the biological data, we present an agent-based model for these stripes that accounts for the behavior of all three types of pigment cells. The development of both wild-type and mutated patterns will be discussed, as well as the effects of fish domain growth on the scale of long-range interactions. Joint work with Bjorn Sandstede.

Shuangquan Xie

Hopf bifurcation for two dimensional Schnakenburg Model

We consider the stability of a single spot solution to the Schnakenburg Model in a two-dimensional disk domain. For large values of the reaction-time constant t, this spot can undergo two different types of Hopf bifurcations. The first Hopf bifurcation induces height oscillation. The second Hopf bifurcation induces a circular motion of the spot position. We use formal asymptotics to delieate the two different regimes. For the rotating spot, we derive a reduced PDE-ODE system to characterize the dynamics and compute the radius and the speed of its rotation by solving the reduced system.

Xiaofeng Ren

The impact of the domain boundary on an inhibitory system: boundary half discs in stationary assemblies

The nonlocal geometric variational problem derived from the Ohta-Kawasaki diblock copolymer theory is an inhibitory system with self-organizing properties. The system can prevent a disc from drifting towards the domain boundary. This raises the question whether a stationary set may have its interface touch the domain boundary. It is proved that a small, perturbed half disc exists as a stable stationary set, where the circular part of its boundary is inside the domain, as the interface, and the almost flat part of its boundary coincides with part of the domain boundary. The location of the half disc depends on two quantities: the curvature of the domain boundary, and a remnant of the Green's function after one removes the fundamental solution and a reflection of the fundamental solution. The notion of reflection here is an interesting new concept that generalizes the familiar notions of mirror image and circle inversion. Our analysis of a boundary half disc leads to constructions of stationary assemblies with both interior discs and boundary half discs.

Matthias Winter

Existence and Stability of Spike Clusters for Biological Reaction-Diffusion Systems

We study the existence and stability of spike clusters for biological reaction-diffusion systems with two small diffusion constants. In particular we consider the Gierer-Meinhardt system with a precursor gradient. In a spike cluster the spikes converge to the same limiting point. We will present results on the asymptotic behaviour of the spikes including their shapes, positions, and amplitudes. We will also compute the asymptotic behaviour of the eigenvalues. Such systems and their solutions play an important role in biological modeling to account for the bridging of lengthscales, e.g. between genetic, nuclear, intra-cellular, cellular and tissue levels, or for the hierarchy of biological processes, e.g. first a large-scale structure appears and then it induces patterns on a smaller scale. This joint work with Juncheng Wei.

Juncheng Wei

On Type II blow up solutions for harmonic map flow

We consider the following harmonic map flow equation:

$$u_t = \Delta u + |\nabla u|^2 u$$

where $u : \Omega \to S^2, |u| = 1$ and Ω is a general two-dimensional domain. We construct in general (non-symmetric) domain and general (non-symmetric) initial condition a Type II blow-up with blowing rate at $(T-t)/(\log^2(T-t))$. Furthermore we show that this blow-up is generic, universal and stable. We will also discuss other types of solutions: infinite-time blow-ups, bubble-trees and reverse bubbles. (joint work with M. del Pino and J. Davila)

Participant list

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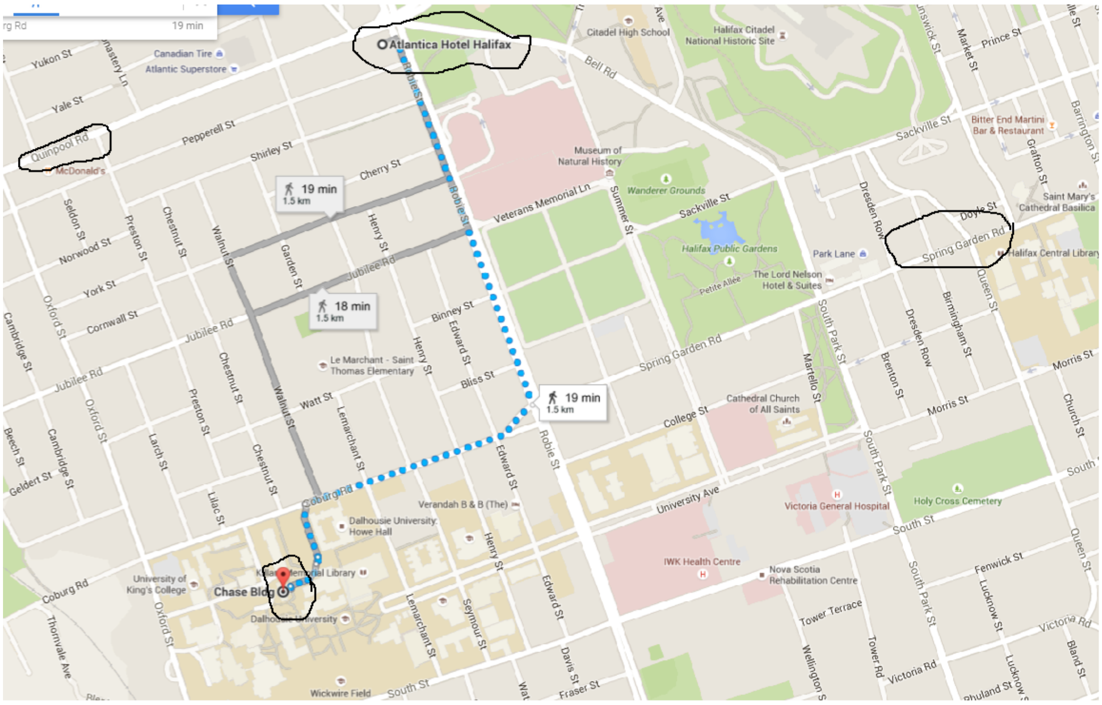
Local information

Most of the participants are staying at Atlantica Hotel, 1980 Robie Street, Halifax; others are staying at Dalhousie university residence, Howe Hall, 6230 Coburg Road.

To get to your hotel/residence from the airport, you have three choices: either taxi (about $70) or a bus (< $5) or a shuttle. I recommend using East Shuttle company, which is as convenient as taxi but much cheaper: http://www.eastshuttle.com/. They charge $35 for a single person, $25 for two and $20 for three or more going at the same time. If you wish to use EAST shuttle, please email Trevor, trevor.d.maclean@gmail.com (please do so in advance) with the following information: (a) Your name/email; (b) The flight number and arrival time to Halifax; (c) Your hotel (either Atlantica or Howe hall). They take you right there.

Here is a map to get from Atlantica hotel to the workshop, which takes place at Chase building, room 319.



Places to eat near workshop:
University club (opens mon-fri)
Couburg cafe (6085 Coburg Rd)
Spring garden road
Quinpool rd