Responses to anonymous feedback

Nov. 20, 2014

1. Question: Will there be plotting on the final?

   Yes.

2. Question: Will the final be biased towards the latter half of the course?

   It is likely, but I strongly encourage you to be clear on **all** of the topics that we have covered – **everything** is fair game.

3. Comment: Make a list of all of the derivations.

   You can of course do this when you go over your notes when studying for the final. Some (**not all**) of the derivations include: obtaining the integrating factor, justification of separation of variables, Abel's formula, formula for variation of parameters, and Laplace transforms of periodic functions and convolutions.

4. Comment: Would like more bonus question opportunities.

   There is still one unsolved bonus problem. I have not decided yet whether there will be a bonus question on the final.

Oct. 23, 2014

1. Comment: Not enough time for the quiz.

   This one looked a little hit or miss - some did fine, some not so much. Time constraints are unavoidable, but it could have been that I was asking for a little too much on this one. I suspect what took most of you a long time was differentiating the y_p guess twice and substituting into the ODE on # 1. Two things that may have been helpful there are 1) the expression $(uv)'' = uv'' + 2u'v' + vu''$, and 2) that the terms proportional to x must cancel. In any case, time constraints are a reality, and if you feel that these quizzes are a bit too short for you, I strongly recommend that you do more practice problems to gain familiarity with the material. I do expect that as the course goes on, and you become better acquainted with the mechanics, that your speed will improve. At the very least, I do not feel that the quiz was unreasonable, as there were people that did fine.

2. Question: What is the meaning of x^s in the "guess" for method of undetermined coefficients?

   The power s is the **smallest** integer such that no term in the guess for the particular solution is a multiple of the homogeneous term. A couple of simple examples:

   $$y'' + y = x^2 \sin x\,.$$

   We always find the homogeneous solutions first, so we calculate $y_1 = \cos x$, $y_2 = \sin x$. Based on the right-hand side, you might guess

   $$y_p = (a_2x^2 + a_1x + a_0)\cos x + (b_2x^2 + b_1x + b_0)\sin x\,.$$

However, you see that $a_0 \cos x$ and $b_0 \sin x$ are multiples of y_1 and y_2, respectively. Therefore, we must multiply our guess by x (here, $s = 1$):

$$y_p = x[(a_2x^2 + a_1x + a_0)\cos x + (b_2x^2 + b_1x + b_0)\sin x] \, .$$

Notice that now no term in y_p is a multiple of the homogeneous term. Next:

$$y'' - 2y' + y = e^x \, .$$

Here, we have $y_1 = e^x$ and $y_2 = xe^x$ because we have repeated roots in the characteristic equation. So our guess would be $y_p = Ax^2e^x$ (here $s = 2$). Multiplying e^x by x would not work, since that would result in a multiple of y_2.

3. Why are we allowed to remove the homogeneous solution in the particular solution?

   Remember that the particular solution y_p is only part of the general solution y, which is actually what is important. The general solution is a sum of the homogeneous and particular solutions; that is, $y = c_1y_1 + c_2y_2 + y_p$, where c_1 and c_2 are arbitrary constants to be obtained from applying initial conditions. If we have $y_p = \tilde{y}_p + c_3y_1$, then the general solution would simply be written $y = (c_1 + c_3)y_1 + c_2y_2 + \tilde{y}_p$. Therefore, we may take $c_3 = 0$ without loss of generality.

4. Question: Why can we take the constant of integration in variation of parameters to be zero?

   For the same reason as above. The general solution is written $y = c_1y_1 + c_2y_2 + u_1(t)y_1 + u_2(t)y_2$. If there are any constants in u_1 or u_2, they may be absorbed into c_1 and c_2. Therefore, we may just take them to be zero without loss of generality.

5. Comment: Turning pages back and forth when using the back side of the page on the quiz is time-consuming.

   I would just copy down the equation and/or relevant details onto the back - even for a 15 minute quiz, this does not seem like it should be too bad. I put all the questions on the front side because I do not know how much space you will need for each question.

Oct. 16, 2014

1. Question: Why are there three distinct roots to the equation $r^3 - 1 = 0$, and only one to $(r-1)^3 = 0$?

   I would just suggest looking at the analogous second order equations $r^2 - 1 = 0$ and $(r-1)^2 = 0$. Clearly the roots of the former are -1 and 1, while they are $1, 1$ for the latter. Remember that the number of roots is equal to the order of the polynomial.

2. Question: Do you think people who test poorly do not deserve a future in academics?

   No, of course not. In fact, many upper-level courses and most graduate courses are predominantly, if not completely, homework and project-based. The reason lower-level courses are usually not run that way is because homework solutions are easily obtained through solution manuals, Maple, Mathematica, etc. Weighting homeworks heavily therefore makes very little sense both from an evaluation and learning perspective. I think the system we have right now where quiz questions come from homework questions is quite fair. The other thing is, the more tests you take, the better you will become at taking them. Feeling extra-confident in the material will also help.

3. Comment: Listening while copying notes is a little difficult.
   Indeed it is, but it is a good skill to have and one that you will develop as time goes on. There generally should not be too much overlap, because I do not explain things while I write. If you would like something repeated during class, please let me know.

4. Comment: Do not skip steps and explain where things come from.
   I think I generally do try to explain where things come from. The steps that I skip are usually things like very involved integrations that I would not test you on (you should know by now which integration techniques you will need), or solving algebraic systems that you already know how to do. Please stop me in class if you need clarification.

Oct. 9, 2014

1. Comment: Do one or two examples first before letting you do some on your own.
   I will do that.

2. Comment: Examples with lots of algebra make things more confusing – a concise set up of the problem along with notes on the techniques involved would be better than working out examples in full.
   This is actually the way that I used to try to do things, except I found that students did not like examples with skipped steps, or ones that were not fully worked out. While providing only "skeleton solutions" does make room for more examples and more time for explanations, I suspect that the majority of students would prefer fully worked out examples. There sometimes also might be technical details that come up while the solution is being worked out, and I would like to highlight those as well.

Oct. 2, 2014

1. Comment: Pacing is too fast.
   Please stop me in class if I am going to fast.

2. Question: For the third order example in class ($y''' - 3y'' - 4y' + 12y = 0$), would it not be faster to factor the characteristic equation instead of doing synthetic division etc.?
   Yes of course – if you can spot the solutions, then you are done. However, cubics are not always easy to factor, or else we could always solve them. Consider the equation $r^3 + 8r^2 + 32r + 51$, where I tell you that $r = -3$ is a solution. Factoring out $r + 3$ would probably be the best way to go here, as spotting the other two solutions would be nearly impossible.

3. Comment: Be more specific when doing proofs and derivations – they just look like a bunch of equations with skipped steps.

   Okay. If you feel that I have skipped too many steps, or are unsure as to why certain things are being done, please stop me. I will always think that what I am doing makes sense, so I will not know that something is unclear unless you tell me. Please come see me if you have specific questions.

4. Comment: More examples, and if an example is taken from the textbook, note the page.

   When I have done examples from the book, I have written down the page that it came from. I usually take examples from outside the book. More examples would be ideal, but there is a certain amount of material that needs to be covered. If you would like to see more examples, the homework and suggested problems are good things to try.

Sept. 25, 2014

1. Comment: Pacing is too fast.

   Again, please stop me in class if I am going to fast.

2. Comment: Explain the quiz right after.

   That tends to cut into lecture time, but perhaps it would be a good investment. I will keep that in mind.

3. Comment: Not enough time for the quiz.

   I will write quizzes that I think should be doable in 15 minutes, but will play individual quiz durations by ear. The hope is that, by doing the homework, you become familiar enough with the material to be able to complete the quiz sufficiently quickly.

4. Comment: Include midterm-level questions on the homework.

   On Homework # 1, I thought 2, 3, and 5 would have been suitable midterm-like questions. Also, on midterms, I may ask for derivations that we covered in class, which I generally do not ask on homeworks since they are in your notes.

5. Comment: Out of practice with integration.

   I am assuming that you are proficient with basic integration techniques (in particular, partial fractions, substitution, and integration by parts). I can suggest that you try the practice problems posted to get into the practice of integration, or you can do problems out of your calculus textbook. As a side note - I did not ask anything about integration by parts on the first quiz, because I know that some of you have not learned it. But it will show up eventually.

6. Comment: Post practice problems.

   I do post them - they are updated periodically.

7. Comment: Start with easy examples first.

   I will do that.

8. Question: This equation $y' + p(x)y = q(x)$ is linear. Can you explain it in other terms?

   This equation is linear because it is linear in y and y'. The operator $d/dx + p(x)$ is a linear operator, which has the properties that we went over in class for second order linear operators. Perhaps a good analogy would be to look at the equation $\mathbf{A}\vec{x} + p\vec{x} = \vec{q}$, where $\mathbf{A}$ is a matrix, p is a constant scalar, and $\vec{q}$ is a constant vector (this is kind of convoluted, because the $p\vec{x}$ term could be combined with the $\mathbf{A}\vec{x}$ term by redefining $\mathbf{A}$). For a linear ODE, we replace $\mathbf{A}$ by a derivative, the constant p by $p(x)$ (independent of y), and $\vec{q}$ by $q(x)$.

9. Comment: Faster way to go from Bernoulli's equation to linear equation.

   Terrific. We discussed this in class – I just wanted to emphasize again that if you ever think of a better or more clever way to think about something, please let me know and I will share it with the class. Or you can go up and present it – that would be good practice for you and make the class much more engaging on the whole.

Sept. 18, 2014

1. Comment: The pacing is too fast.

   I have tried to slow down, and it seems that there are fewer people this week that think things are too fast. Again, please let me know in class when something is not clear. If you are uncomfortable doing that, please make use of this forum to ask your question(s).

2. Comment: Post practice problems from the textbook.

   I will do that.

3. Comment: Examples make things clearer.

   I agree. However, when I introduce a new topic, I have to define terminology and give background theory. This theory is also important. While the emphasis of this class is on methods for solving ODE's, I also expect you to understand why these methods work. For example, you should understand how to solve separable equations without simply manipulating the dx's and dy's, and how to derive an integrating factor for linear equations (without memorizing).

4. Comment: Product rule and integration by parts are unclear.

   It seems that some of you have not seen the product rule and/or integration by parts. These two concepts are crucial (not just for this class). I won't be going over this in class, so please come see me if you need help in this regard. Reading relevant sections of any first-year calculus textbook may also help.

5. Question: It seems that if a solution to an ODE has a negative quantity under a square root (that is, the solution becomes complex), we say that the solution "does not exist." But even if a solution is complex, can it not still have physical significance?

   Yes and no. Physical quantities that we can measure must be real. A complex amount of money or bacteria makes no sense. Neither do complex locations and velocities. So when we say that the solution does not exist, we mean that no real solutions exist. Note that if the solution is complex, the ODE must be thought of as a system of two ODE's (one for the real part, one for the imaginary part).

However, complex quantities can yield physically relevant information. For example, the wave function $\psi(x,t)$ of a particle is complex, but its square integral $\int_A |\psi|^2\,dx$ is real, and (properly normalized) gives the probability that the particle is inside the interval A at time t. For another example, the imaginary part of the complex velocity potential for a steady two-dimensional flow of an incompressible and inviscid fluid is referred to as the stream function. Contours of this function represent the direction of the paths of fluid particles transported by the flow.

6. Comment: Projector is better than the board.
   I am told that the projector has been fixed.

7. Question: When computing the integrating factor, why are we allowed to set the integration constant to be 0 without loss of generality?
   The **general** solution of the linear first order equation

$$\frac{dy}{dt} + p(t)y = q(t)\,, \qquad y(t_0) = y_0\,,$$

is given by

$$y(t) = \frac{\int u(t)q(t)\,dt + c}{u(t)} \equiv \frac{R(t)+c}{u(t)}\,,$$

where $u(t) = e^{\int p(t)\,dt}$ is the integrating factor computed by setting the integration constant to 0. Not setting it to 0 would be equivalent to multiplying $u(t)$ by an arbitrary constant. Let us call this constant A. In this case, the general solution would be given by

$$y(t) = \frac{AR(t)+c}{Au(t)}\,. \tag{1}$$

Now we apply initial conditions: $y(t_0) = y_0 = (AR(t_0)+c)/(Au(t_0))$, which yields $c = A[y_0 u(t_0) - R(t_0)]$. Substituting this expression for c back into (1), we see that the factor A cancels out. Therefore, we may take $A = 1$ without loss of generality. That is, we may take the integration constant for the integrating factor to be 0, or any value that is convenient.

8. Question: How did you do the synthetic division example in class?
   Synthetic division is something that I learned in high school, and it has worked so well and unfailingly that I have never questioned how it works. As such, I have no idea how it works. You can read the Wikipedia page `http://en.wikipedia.org/wiki/Synthetic_division` to see an example, or just use regular polynomial division.

9. Comment: Do one example that covers all concepts instead of many examples, each of which covers only a few.
   Aside from the fact that such examples are difficult to cook up, illustrating too many concepts with one example can be confusing. It seems that most students prefer multiple examples, so I will continue along those lines.

10. Comment: Start with trivial examples.

    I will keep this in mind.

11. Comment: Post syllabus online.

    The list of chapters of the book that we will cover (time permitted) is online (the very first link). It is very difficult to provide a week-by-week forecast of topics to be covered.

12. Comment: Go over the rescaling examples again.

    I would rather not spend too much more time on this, but I have written out the two examples that we did in class and posted it on the website. Rescaling does take a bit of practice.

13. Comment: Clarify the MFPT example.

    Please come see me if you have questions regarding the MFPT example.

Sept. 11, 2014

Thanks very much for the feedback. It was quite helpful, and I will try to make the necessary adjustments.

1. Comment: Go slower.

   I will go slower. Please stop me in class if something is not clear, even if you are not sure what your question is.

2. Comment: Too many definitions, theory, and digressions.

   The definitions and theory are required material, and I decided to put all of it at the beginning so that the rest of the lectures flow better. Lectures starting from here will be much more example-rich. As for the digressions, since this is your first class on differential equations, which is an extremely broad and rich field, I think it benefits you to gain some perspective on where this class fits in the bigger picture. They are things that I wish I had seen when I took ODE's. However, in accordance with the feedback, I will narrow the focus in the future.

3. Comment: Not enough theory before doing examples.

   Sorry...you are in the minority...

4. Comment: Did not learn Taylor series in first year calculus.

   A prerequisite for this course is MATH 1010.03, the online description of which includes Taylor series. Since it appears that many of you did not see it in your first year classes, I will go over it quickly next class. You can also read up on it in your calculus textbook. To answer a related question, I can only read online descriptions of your prerequisite courses - it is up to you to let me know (as you did here) if a concept is not clear to you.

5. Comment: Put lecture notes online.

   If I post the notes online, you will have less incentive to copy notes in class, or even to come to class. Studies have shown that students learn better when they copy notes, because it forces them to engage with the material. As such, I would rather not post them for now. Also, I do not use BB Learn - the course website is `http://www.mathstat.dal.ca/~tzou`

6. Comment: Explain physics concepts more clearly.

   I have indeed assumed some knowledge of physics concepts so far in some of the motivating examples. I will make sure to explain it more clearly when they come up again.

7. Comment: Name the sections of the textbook we are covering.

   Some of the introductory material was not covered in the book, but otherwise, I thought I was doing that (separable equations §1.4, linear equations §1.5 etc...).

8. Comment: Why do the MFPT example if it was not in the book?

   I was not planning on doing the derivation, but a very good question was asked about it. I thought it was quite a rich example from which you could learn quite a bit. In particular, you now know how ODE's can be derived, and that their solutions represent very relevant and physical quantities. Moreover, random walk formalisms are used in many very different fields, and rescaling of equations is also a good thing to learn.

9. Comment: Do we need to know the MFPT derivation?

   No. It was done for your general interest/knowledge. Or just knowledge...

10. Comment: Do we need the textbook?

    I highly recommend it. It can help clarify concepts that were not clear in class. Some homework questions will be assigned from the textbook. There is one on reserve in the library. Please let me know if one is not enough.

11. Comment: More examples, less theory.

    That is the plan. I just wanted to get the theory and definitions out of the way first.

12. Comment: Are the examples out of the textbook, and if so, state which page.

    I sometimes try to do examples not in your textbook so that you can see more examples in total. Of course, I highly recommend that you follow along with the textbook as we go. This way, when I do an example from the book, you will know which page it is.