(1)

Homework #1

(1) p.43 #4

$$(1+x)\frac{dy}{dx} = 4y$$

$$\int \frac{dy}{y} = \int \frac{4}{1+x}\,dx$$

$$\log y = 4\log(1+x) + c$$

$$= \log\left(\tilde{c}\,(1+x)^4\right)$$

$$\Rightarrow \boxed{y = \tilde{c}\,(1+x)^4}$$

p.43 #17

$$y' = 1+x+y+xy$$

$$= (x+1)(y+1)$$

$$\int \frac{dy}{y+1} = \int dx\,(x+1)$$

$$\log(y+1) = \frac{(x+1)^2}{2} + C \Rightarrow y+1 = Ce^{\frac{(x+1)^2}{2}}$$

$$\boxed{y = -1 + Ce^{\frac{(x+1)^2}{2}}}$$

(2)

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = 1$$

$$\int \frac{dy}{y^3} = \int x \frac{dx}{\sqrt{1+x^2}}$$

$$\frac{y^{-2}}{-2} = \sqrt{1+x^2} + c$$

redefined...

$$\frac{1}{y^2} = -2\sqrt{1+x^2} + c$$

$$y = \frac{1}{\sqrt{c - 2\sqrt{1+x^2}}}$$

$$y(0) = 1 \Rightarrow \frac{1}{\sqrt{c-2}} = 1 \Rightarrow c = 3$$

$$\boxed{y(x) = \frac{1}{\sqrt{3 - 2\sqrt{1+x^2}}}}$$

require

$$3 - 2\sqrt{1+x^2} > 0$$

$$\frac{3}{2} > \sqrt{1+x^2}$$

$$\frac{9}{4} > 1 + x^2$$

$$\frac{5}{4} > x^2$$

$$\Rightarrow \boxed{|x| < \frac{\sqrt{5}}{2}}$$

← interval of existence

3

$$\frac{dy}{dx} = y^2(x+2) \qquad y(0) = 1$$

$$\int \frac{dy}{y^2} = \int (x+2)\,dx$$

$$-\frac{1}{y} = \frac{(x+2)^2}{2} + c$$

$$y^{-1} = -\frac{(x+2)^2}{2} + c$$

$$y = \frac{1}{C - \frac{(x+2)^2}{2}} = \frac{2}{C - (x+2)^2}$$

redefined C ...

$$y(0) = \frac{2}{C-4} = 1$$

$$\Rightarrow \quad C - 4 = 2 \quad \Rightarrow \quad C = 6.$$

$$\boxed{y = \frac{2}{6 - (x+2)^2}}$$

extremum when $dy/dx = 0$

this occurs when $x = -2$

$(y \neq 0)$. Is this a minimum or maximum?

notice that

$$y' = \cancel{y^2}(x+2)y^2$$

$$y'' = y^2 + (x+2)2yy'$$

$$= y^2 > 0$$

$$\Rightarrow y'' > 0 \Rightarrow \text{minimum}$$

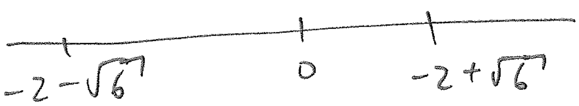
denominator 0 when

$$6 - (x+2)^2 = 0$$

$$\Rightarrow x^2 + 4x - 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16+8}}{2} = -2 \pm \frac{\sqrt{24}}{2}$$

$$= -2 \pm \sqrt{6}$$



since IC posed at $x=0$,

interval of existence is

$$\boxed{-2-\sqrt{6} < x < -2+\sqrt{6}}$$

4 p.54 #4

$$y' - 2xy = e^{x^2}$$

int. factor is $u(x) = e^{-x^2}$

$$\frac{d}{dx}\left(e^{-x^2} y\right) = 1$$

$$e^{-x^2} y = x + c$$

$$\boxed{y = x e^{x^2} + c e^{x^2}}$$

p.54 # 7

$$2xy' + y = 10\sqrt{x}$$

$$y' + \frac{1}{2x} y = \frac{10\sqrt{x}}{2x}$$

integrating factor $e^{\int \frac{1}{2x} dx} = e^{\frac{1}{2}\log x} = \sqrt{x}$

$$\frac{d}{dx}(\sqrt{x}\, y) = \frac{10x}{2x} = 5$$

$$\Rightarrow \sqrt{x}\, y = 5x + c$$

$$\boxed{y = 5\sqrt{x} + \frac{c}{\sqrt{x}}}$$

p. 54 #16

$$y' = (1-y)\cos x, \quad y(\pi) = 2$$

$$y' + (\cos x) y = \cos x$$

int. factor $u(x) = e^{\int \cos x\, dx} = e^{\sin x}$

$$\frac{d}{dx}(e^{\sin x} y) = e^{\sin x} \cos x$$

$$e^{\sin x} y = e^{\sin x} + c$$

$$\boxed{y = 1 + Ce^{-\sin x}}$$

(5) $\dfrac{dy}{dt} - \dfrac{1}{2}y = 2\cos t \qquad y(0) = a$

int factor $\quad u(t) = e^{-\frac{1}{2}t}$

$$\frac{d}{dt}\left(e^{-\frac{1}{2}t}y\right) = 2\cos t\, e^{-\frac{1}{2}t}$$

now

$$I = \int \cos t\, e^{-\frac{1}{2}t}\, dt = ?$$

$u = \cos t \qquad dv = e^{-\frac{1}{2}t}dt$

$du = -\sin t\, dt \qquad v = -2e^{-\frac{1}{2}t}$

$$I = -2\cos t\, e^{-\frac{1}{2}t} - 2\int \sin t\, e^{-\frac{1}{2}t}\, dt$$

$u = \sin t \qquad dv = e^{-\frac{1}{2}t}dt$

$du = \cos t\, dt \qquad v = -2e^{-\frac{1}{2}t}.$

$$= -2\cos t\, e^{-\frac{1}{2}t} - 2\left[-2\sin t\, e^{-\frac{1}{2}t} + 2\int \cos t\, e^{-\frac{1}{2}t}dt\right]$$

$$= -2\cos t\, e^{-\frac{1}{2}t} + 4\sin t\, e^{-\frac{1}{2}t} - 4I$$

$$\Rightarrow 5I = -2\cos t\, e^{-\frac{1}{2}t} + 4\sin t\, e^{-\frac{1}{2}t}$$

$$I = -\frac{2}{5}\cos t\, e^{-\frac{1}{2}t} + \frac{4}{5}\sin t\, e^{-\frac{1}{2}t}.$$

$$\Rightarrow e^{-\frac{1}{2}t} y = \left[-\frac{4}{5}\cos t + \frac{8}{5}\sin t\right] e^{-\frac{1}{2}t} + c$$

$$y = -\frac{4}{5}\cos t + \frac{8}{5}\sin t + c e^{t/2}$$

$$y(0) = a = -\frac{4}{5} + c \Rightarrow c = a + \frac{4}{5}$$

$$y(t) = \frac{-4}{5}\cos t + \frac{8}{5}\sin t + \left(a + \frac{4}{5}\right)e^{t/2}$$

$a = \frac{-4}{5}$: y remains finite for all t

$a < \frac{-4}{5}$: $y \to -\infty$ as $t \to \infty$

$a > \frac{-4}{5}$: $y \to +\infty$ as $t \to \infty$.