MATH 2120 – Homework 3 (due Thursday October 16, 2014 before class)

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1. **Alternate way of obtaining the second solution in the case of repeated roots.** Consider the second order linear constant coefficient equation

$$y'' + 2ay' + a^2 y = 0\,. \tag{1}$$

   (a) Show that the corresponding characteristic equation has repeated roots $r_1 = r_2 = -a$ so that $y_1 = e^{-at}$ is a solution of (1).

   (b) Use Abel's formula to show that the Wronskian $W(y_1, y_2; t)$ of any two solutions of (1) is $W = c_1 e^{-2at}$.

   (c) Letting $y_1 = e^{-at}$, use the result from part (b) to derive a differential equation for y_2. Solve this equation to show that $y_2 = te^{-at}$.

2. **Another way of obtaining the second solution in the case of repeated roots.** For the ODE $ay''+by'+cy = 0$, suppose that r_1 and r_2 are two distinct roots of the corresponding characteristic equation. Thus, $y_1 = e^{r_1 t}$ is one solution. The other (linearly independent) solution can be written as

$$y_2 = \frac{e^{r_2 t} - e^{r_1 t}}{r_2 - r_1}\,. \tag{2}$$

   Now let $r_2 = r_1 + \varepsilon$ in (2). Show that $\lim_{\varepsilon \to 0} y_2 = te^{r_1 t}$. That is, as the two roots coalesce into a repeated root, the second solution approaches $te^{r_1 t}$.

3. Section 2.2 (p.124): #41, #43.

4. Section 2.3 (p.134): #22, #27, #29*.

   * You can use the method from class for finding roots of unity if you like – it is much faster

5. Solve the differential equation

$$\frac{d^2x}{dt^2} + 4x = 0,$$

   for the following initial conditions, and write in phase-amplitude form $x(t) = C\cos(\omega t - \alpha)$ for constants C, ω, and α:

   (a) $x(0) = 1, \quad x'(0) = -2$

   (b) $x(0) = -1/2, \quad x'(0) = \sqrt{3}$

   (c) $x(0) = 1/2, \quad x'(0) = \sqrt{3}$

   (d) $x(0) = -1, \quad x'(0) = -2$

6. Section 2.4 (p.147): # 17.