MATH 2120 – Homework 4 (due Thursday October 23, 2014 before class)

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1. Section 2.5 (p.161) #4, #5, #8, #16*, #21, #23, #25

   [Hint for #5: show using Euler's formula that $\sin^2 x = (1 - \cos(2x))/2$.]

   * **Only** write down the general form of the particular solution; **do not** solve for the constants.

2. Use the method of undetermined coefficients to solve the initial value problem

$$y'' + y = \begin{cases} t\,, & 0 \le t \le \pi\,, \\ \pi e^{\pi - t}\,, & t > \pi\,, \end{cases} \qquad y(0) = 0, \quad y'(0) = 1\,.$$

   Note that $e^{\pi-t} = e^{\pi}e^{-t}$. Your solution should be of the form

$$y(t) = \begin{cases} y_l(t)\,, & 0 \le t \le \pi\,, \\ y_r(t)\,, & t > \pi\,. \end{cases}$$

   To find $y_l(t)$, solve $y'' + y = t$ with $y(0) = 0$ and $y'(0) = 1$. Remember to apply the initial conditions only after having found the full general solution, which includes the particular solution. To find $y_r(t)$, first find the general solution of $y'' + y = \pi e^{\pi - t}$. To determine the two unknown constants, apply continuity of y and y' at $t = \pi$. That is, $y_r(\pi) = y_l(\pi)$ and $y_r'(\pi) = y_l'(\pi)$. You can think about why y and y' must be continuous.

3. Variation of parameters states that the particular solution of $y'' + p(t)\,y' + q(t)\,y = f(t)$ is given by

$$y_p(t) = y_1(t)\int \frac{-y_2(t)f(t)}{W(y_1, y_2\,; t)}\,dt + y_2(t)\int \frac{y_1(t)f(t)}{W(y_1, y_2\,; t)}\,dt\,, \tag{1}$$

   where $W(t; y_1, y_2)$ is the Wronskian of the two linearly independent homogeneous solutions $y_1(t)$ and $y_2(t)$. Use result (1) to find the particular solution of

$$y'' + y = \sin t\,.$$

   Be sure to identify any homogeneous terms that may appear in your expression for the particular solution. Verify your answer using the method of undetermined coefficients. [I will give you (1) on exams, but please make an effort to learn the derivation, as derivations are in general good things to understand.]