MATH 2120 – Homework 6 (due Thursday November 27, 2014 before class)

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1. Section 4.4 (p.304) #10, #21. For #21, explain why the Laplace transform exists (important point).

2. Section 4.5 (p.314) #4, #21.

3. Section 4.6 (p.324) #21(b), #22.

4. Section 5.1 (p.335) #2.

5. Section 5.4 (p.378) #4, #6, #8, #11. Please draw the phase portraits by hand.

6. Consider the initial value problem

$$x'' + x = [u(t-2\pi) - u(t-4\pi)]\sin t \equiv f(t)\,, \qquad x(0) = 0\,, \quad x'(0) = 0\,, \tag{1}$$

where $u(t-a)$ is the Heaviside step function, taking on the value of 0 when $t < a$ and 1 when $t \geq a$.

(a) Find the Laplace transform of $f(t) = [u(t-2\pi) - u(t-4\pi)]\,\sin t$. The following may be useful:

$$\sin(kt - n\pi) = (-1)^n \sin(kt)\,, \qquad \cos(kt - n\pi) = (-1)^n \cos(kt)\,.$$

(b) Solve (1) using Laplace transforms. The following may be useful:

$$\mathcal{L}\left\{\frac{1}{2k^3}\left(\sin(kt) - kt\cos(kt)\right)\right\} = \frac{1}{(s^2+k^2)^2}\,.$$

(c) **Briefly** explain the behavior of the solution in each of the three relevant time intervals in relation to the forcing function $f(t)$ (and initial conditions, if applicable). Is there resonance? If so, does the amplitude of oscillations approach ∞ as $t \to \infty$? Why or why not?

(d) What is the amplitude of oscillations for $t > 4\pi$?