5.4 #8 on hw

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix} \vec{x} \qquad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

$$\det \begin{pmatrix} 1-\lambda & -5 \\ 1 & -1-\lambda \end{pmatrix} = 0 \implies \lambda^2 - 1 + 5 = 0$$

$$\lambda = \pm 2i$$

for $\lambda = 2i$,

$$\begin{pmatrix} 1-2i & -5 \\ - & - \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = 0.$$

$$(1-2i)\xi_1 - 5\xi_2 = 0.$$

$$\xi_1 = 5, \quad \xi_2 = 1-2i$$

then

$$\vec{\xi} e^{\lambda t} = \begin{pmatrix} 5 \\ 1-2i \end{pmatrix} e^{i2t}.$$

$$= \begin{pmatrix} 5\cos 2t + 5i \sin 2t. \\ \cos 2t + 2\sin 2t - 2i\cos 2t + i \sin 2t \end{pmatrix}.$$

$$= \begin{pmatrix} 5\cos 2t \\ \cos 2t + 2\sin 2t \end{pmatrix} + i \begin{pmatrix} 5\sin 2t \\ -2\cos 2t + \sin 2t \end{pmatrix}.$$

2. l.i. sln.

so

$$\vec{x} = c_1 \begin{pmatrix} 5\cos 2t \\ \cos 2t + 2\sin 2t \end{pmatrix} + c_2 \begin{pmatrix} 5\sin 2t \\ -2\cos 2t + \sin 2t \end{pmatrix}.$$

<u>or</u>

$$\vec{\xi}^{+} = \overset{\text{scalar}}{\left(\frac{1+2i}{5}\right)} \begin{pmatrix} 5 \\ 1-2i \end{pmatrix} = \begin{pmatrix} 1+2i \\ \frac{1^2+2^2}{5} \end{pmatrix} = \begin{pmatrix} 1+2i \\ 1 \end{pmatrix}$$

then

$$\vec{\xi}^{+} e^{\lambda_+ t} = \begin{pmatrix} 1+2i \\ 1 \end{pmatrix} e^{2it}.$$

$$= \begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix} + i \begin{pmatrix} 2\cos 2t + \sin 2t \\ \sin 2t \end{pmatrix}.$$

so

$$\vec{x} = k_1 \begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix} + k_2 \begin{pmatrix} 2\cos 2t + \sin 2t \\ \sin 2t \end{pmatrix}.$$

notice

$$\begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix} = \frac{1}{5}\left[\begin{pmatrix} 5\cos 2t \\ \cos 2t + 2\sin 2t \end{pmatrix} - 2\begin{pmatrix} \sin 2t \\ -2\cos 2t \\ + \sin 2t \end{pmatrix}\right]$$

same for the other one.

plot? we know it is a closed orbit, non circular. for what k_1, k_2 is the orbit a tilted ellipse? → obtain tilt angle, major, minor axis.

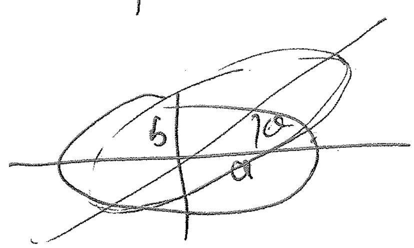
$$x_1 = (k_1 + 2k_2)\cos 2t + (k_2 - 2k_1)\sin 2t.$$

$$x_2 = k_1 \cos 2t + k_2 \sin 2t.$$

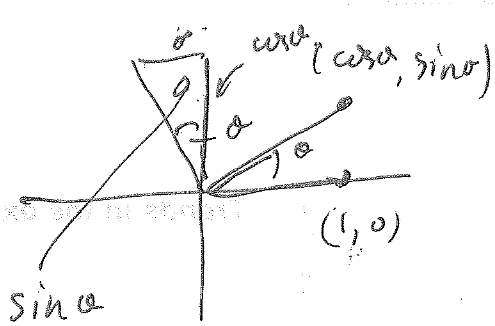
a non-tilted ellipse, has the form.

$$y_1 = a\cos 2t$$

$$y_2 = b\sin 2t$$



apply rotation matrix



$$\underset{=}{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\underset{=}{R}\,\vec{y} = \begin{pmatrix} a\cos\theta\cos 2t - b\sin\theta\sin 2t \\ a\sin\theta\cos 2t + b\cos\theta\sin 2t \end{pmatrix}.$$

we require

$$a\cos\theta = k_1 + 2k_2. \qquad -b\sin\theta = k_2 - 2k_1$$

$$a\sin\theta = k_1 \qquad b\cos\theta = k_2.$$

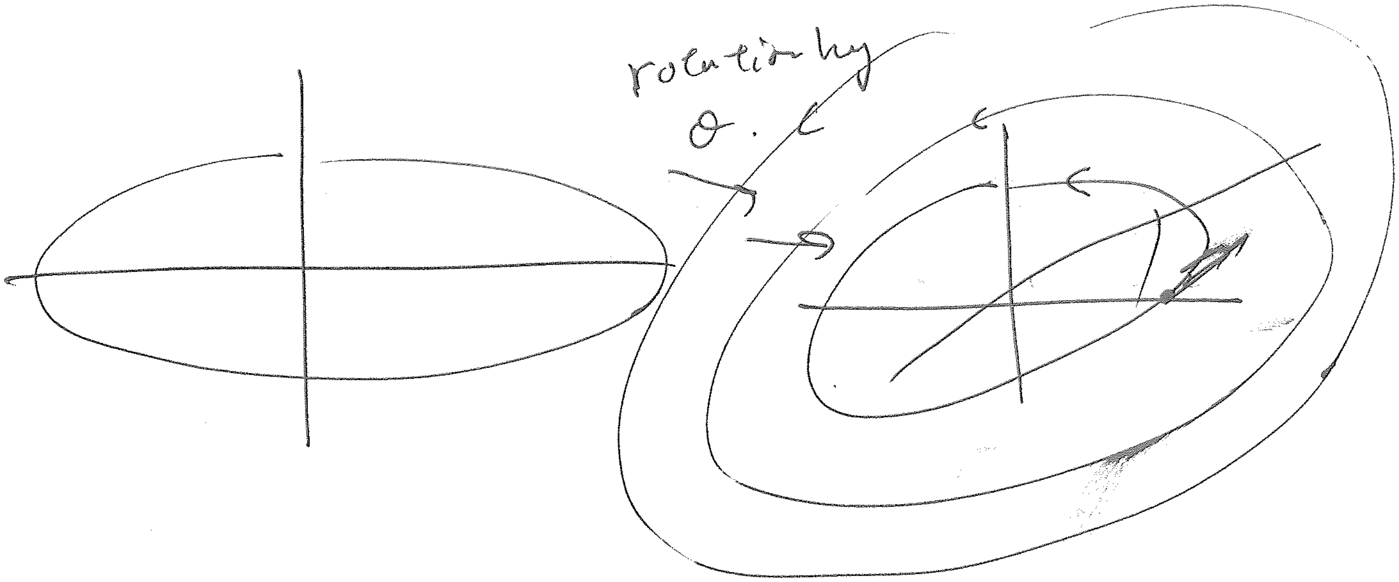
$$\tan\theta = \frac{k_1}{k_1 + 2k_2}, \qquad \tan\theta = \frac{2k_1 - k_2}{k_2}.$$

$$\Rightarrow k_1 k_2 = (k_1 + 2k_2)(2k_1 - k_2).$$

$$\Rightarrow k_1 = \left(\frac{-1+\sqrt{5}}{2}\right) k_2.$$

$$\tan\theta = -2 + \sqrt{5} \Rightarrow \theta = 0.2138.$$

$$\frac{a^2}{b^2} = 6.85.$$



direction?

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$