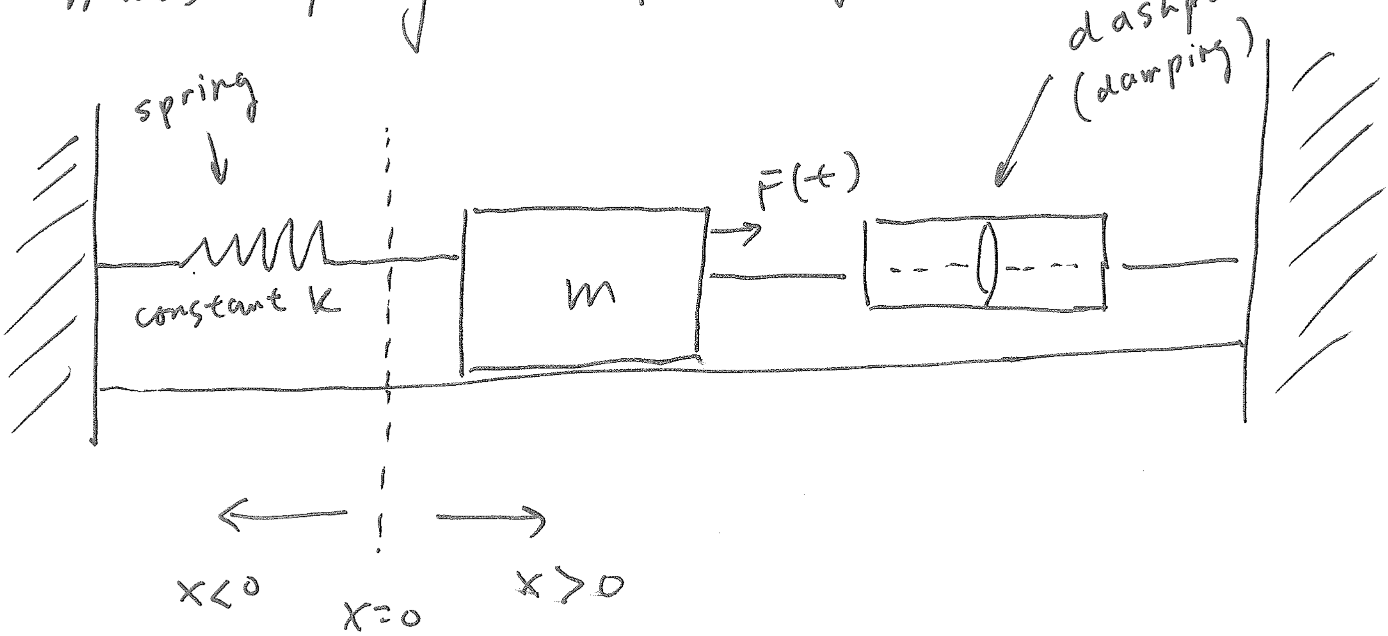
Mechanical vibrations (2.4)

motivating examples

1) mass - spring - dashpot system



three forces :

1) spring : $F_{spring} = -kx$ (restoring force)

$k > 0$

if $x > 0$, $F_{spring} < 0$

if $x < 0$, $F_{spring} > 0$

2) dashpot : $F_d = -c\frac{dx}{dt}$ (damping force $c > 0$)

proportional to velocity.

if $\frac{dx}{dt} > 0$, $F_d < 0$

if $\frac{dx}{dt} < 0$, $F_d > 0$

acts to slow mass down

3) external for $F(t)$

then Newton's Law states

$$F = ma.$$

$$m \underbrace{\frac{d^2x}{dt^2}}_{a} = -c\frac{dx}{dt} - kx + F(t)$$

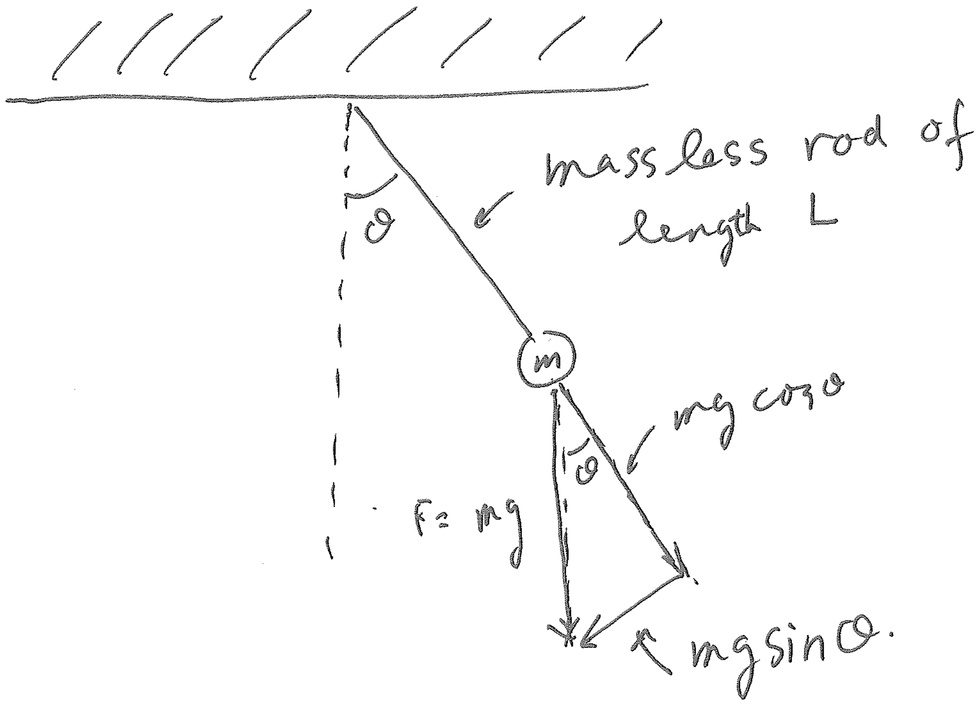
$$mx'' + cx' + kx = F(t)$$

if $F \equiv 0$, then this just the second order linear constant coeff. homogeneous eqn.

2) simple pendulum

(energy formulation in the book)

here we use $F = ma$.



$F = ma$.

$$ma = m\frac{dv}{dt} = \cancel{m\left[L\frac{d\theta}{dt}\right]}$$

$$= m\frac{d}{dt}\underbrace{\left[L\frac{d\theta}{dt}\right]}_{v} = -mg\sin\theta$$

$$\Rightarrow \quad mL \frac{d^2\theta}{dt^2} = -mg\sin\theta.$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta.$$

nonlinear, but if we assume θ is small ($|\theta| \ll 1$), then $\sin\theta \approx \theta$.

then

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0.$$

for damping (e.g. friction), add a

$c\frac{d\theta}{dt}$, $c > 0$ on RHS.