Solution

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MATH 2120 – Quiz 2 Tuesday October 7, 2014

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1. For the first order ODE

$$(3t + y)\, y' = t - 2y\,,$$

make the substitution $v(t) = y(t)/t$ to obtain a separable equation for $v(t)$. Write the equation for v in the form $dv/dt = G(v)/t$. **Do not** solve the equation, and you **do not** need to simplify $G(v)$.

$$\frac{dy}{dt} = \frac{t - 2y}{3t + y} = \frac{1 - 2y/t}{3 + y/t} \quad (2)$$

$$v = y/x \Rightarrow y = xv \ (1) \qquad y' = v + \cancel{x}tv' \ (1)$$

$$v + tv' = \frac{1 - 2v}{3 + v} \Rightarrow tv' = \frac{1 - 2v}{3 + v} - v$$

$$\boxed{\frac{dv}{dt} = \frac{1}{t}\left[\frac{1 - 2v}{3 + v} - v\right]} \quad (1)$$

$$\frac{1}{t}\left[\frac{1 - 5v - v^2}{3 + v}\right]$$

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2. Solve the IVP

$$\frac{d^2x}{dt^2} = 7x\,; \qquad x(0) = \alpha\,, \quad x'(0) = \beta\,.$$

$$\text{c.e.} \quad r^2 = 7 \qquad r = \pm\sqrt{7}$$

$$x = A\cosh\sqrt{7}\,t + B\sinh\sqrt{7}\,t \quad (2) \quad \text{or } Ae^{\sqrt{7}t} + Be^{-\sqrt{7}t}$$

$$x(0) = A = \alpha \Rightarrow A = \alpha$$

$$x'(0) = \sqrt{7}\,B = \beta \Rightarrow B = \beta/\sqrt{7} \quad (2)$$

$$\boxed{x(t) = \alpha\cosh\sqrt{7}\,t + \frac{\beta}{\sqrt{7}}\sinh\sqrt{7}\,t}$$

(1)

$$\text{or} \quad \boxed{\frac{1}{2}\left(\alpha + \frac{\beta}{\sqrt{7}}\right)e^{\sqrt{7}t} + \frac{1}{2}\left(\alpha - \frac{\beta}{\sqrt{7}}\right)e^{-\sqrt{7}t}}$$

3. Consider the equidimensional equation

$$x^2y'' - 3xy' + 3y = 0\,. \qquad (1)$$

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We are given that $y = x$ is a solution of (1) (you need **not** verify this). Use reduction of order to find the other linearly independent solution y_2.

$y_2 = xv$ 2

$y_2' = v + xv'$ 1

$y_2'' = v' + v' + xv''$

$= 2v' + xv''$ 1

$x^2\left[2v' + xv''\right] - 3x(\cancel{v} + xv') + \cancel{3xv} = 0$

$2v' + xv'' - 3v' = 0$

$xv'' = v'$ 2

$\underline{\underline{u = v'}}$

$xu' = u$

$\int \frac{du}{u} = \int \frac{dx}{x}$

okay if they have $\underline{\underline{y_2 = x^3 + cx}}$

$\log u = \log x + c$

$c = 0$ wlog.

$\Rightarrow u = x$ 2

$v = \int u\,dx = \frac{x^2}{2} + c \Rightarrow \boxed{y_2 = x^3}$

$c = 0$ wlog

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