Solution

MATH 2120 – Quiz 3 Tuesday October 16, 2014

1. Find the general solution of the ODE

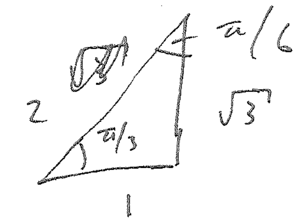
$$y''' + y = 0 .$$

Leave the solution in terms of complex exponentials; **do not** write it in terms of sines and cosines.

2. Find the general solution of the ODE

$$y'' - 2y' + 5y = 0 .$$

Write the solution in terms of sines and cosines.



3. Write the following in phase-amplitude form:

(a) $x(t) = e^{-5t}\left[\sqrt{3}\cos 3t - \sin 3t\right]$

(b) $x(t) = -\cos t + \sin t$

① $y''' + y = 0$ C.E. : $r^3 + 1 = 0$ $r^3 = -1$

$r^3 = e^{i\pi} \Rightarrow r = e^{i\pi/3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$r^3 = e^{i3\pi} \Rightarrow r = e^{i\pi} = -1$

$r^3 = e^{i5\pi} \Rightarrow r = e^{i5\pi/3} = \frac{1}{2} - \frac{i\sqrt{3}}{2}$

$$\Rightarrow \boxed{y = c_1 e^{-t} + c_2 e^{\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)t} + c_3 e^{\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)t}}$$

or can spot solution $r = -1$ so $r+1$ is a factor of r^3+1

$$\begin{array}{r|rrrr} +1 & 1 & 0 & 0 & 1 \\ & & +1 & -1 & +1 \\ \hline & 1 & -1 & 1 & 0 \end{array} \Rightarrow (r+1)(r^2 - r + r) = 0.$$

$r = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}$ etc...

(2)

$$y'' - 2y' + 5y = 0$$

c.e.: $r^2 - 2r + 5 = 0$

$$r = \frac{2 \pm \sqrt{4-20}}{2} = \cancel{2\pm4}\ 1 \pm 2i$$

so $y_1 = e^{(1+2i)t} = e^t [\underbrace{\cos 2t}_{\text{real}} + i \underbrace{\sin 2t}_{\text{imag.}}]$

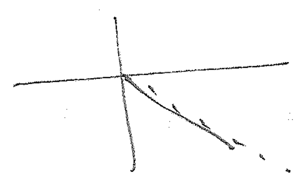
then $\boxed{y = e^t [c_1 \cos 2t + c_2 \sin 2t]}$

(3) $x(t) = e^{-5t}[\sqrt{3}\cos 3t - \sin 3t] = e^{-5t} C\cos(\omega t - \alpha)$

a)

look at periodic part: $\sqrt{3}\cos 3t - \sin 3t$.

$A = \sqrt{3}$, $B = -1$

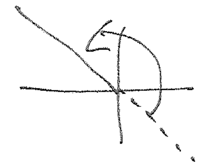


$C = \sqrt{A^2+B^2} = 2$, $\tan^{-1}(B/A) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$.

$\Rightarrow \alpha = -\frac{\pi}{6}$ or $-\frac{\pi}{6} + 2\pi = \frac{11\pi}{6}$.

$$\Rightarrow \boxed{\begin{array}{c} x(t) = e^{-5t} \cdot 2\cos(\cancel{\omega}3t + \frac{\pi}{6}) \\ \text{or} \\ x(t) = e^{-5t} \cdot 2\cos(\cancel{\omega}3t - 11\pi/6) \end{array}}$$

b) $x(t) = -\cos t + \sin t$ $\quad A = -1$, $B = 1$



$C = \sqrt{1+1} = \sqrt{2}$, $\tan^{-1}(B/A) = \tan^{-1}(-1) = -\frac{\pi}{4}$

$\Rightarrow \alpha = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$ $\Rightarrow \boxed{x(t) = \sqrt{2}\cos(t - 3\pi/4)}$