MATH 2120 – Quiz 6 Thursday November 27, 2014

1. Solve

$$x'' + 4x' + 13x = \sum_{n=0}^{\infty}(-1)^n\delta\left(t - \frac{n\pi}{3}\right), \qquad x(0) = x'(0) = 0.$$

Calculate $x(t)$ on the interval

$$\frac{k\pi}{3} < t < \frac{(k+1)\pi}{3}.$$

2. Write the second order equation

$$x'' - x' - 2x = 0, \qquad x(0) = \alpha, \quad x'(0) = \beta,$$

as a system of two first order ODE's in vector-matrix form, **including initial conditions**. Is the origin stable or unstable? Classify the origin.

① $x'' + 4x' + 13x = \sum_{n=0}^{\infty} (-1)^n \delta\left(t - \frac{n\pi}{3}\right)$ $\quad x'(0) = x(0) = 0$

$$(s^2 + 4s + 13)X(s) = \sum_{n=0}^{\infty} (-1)^n e^{-\frac{n\pi}{3}s}$$

$$X(s) = \sum_{n=0}^{\infty} \frac{(-1)^n e^{-\frac{n\pi}{3}s}}{s^2 + 4s + 4 + 9}$$

$$= \frac{1}{3}\sum_{n=0}^{\infty} (-1)^n e^{-\frac{n\pi}{3}s} \frac{3}{(s+2)^2 + 3^2}$$

$\uparrow e^{-2t}\sin 3t.$

$$x(t) = \frac{1}{3}\sum_{n=0}^{\infty} (-1)^n e^{-2\left(t - \frac{n\pi}{3}\right)} \underbrace{\sin\left(3\left(t - \frac{n\pi}{3}\right)\right)}_{(-1)^n \sin(3t)} \cdot u\left(t - \frac{n\pi}{3}\right)$$

$$x(t) = \frac{1}{3}\sum_{n=0}^{\infty} e^{-2t} \sin 3t \; e^{\frac{2n\pi}{3}} u\left(t - \frac{n\pi}{3}\right)$$

on $\frac{k\pi}{3} < t < \frac{(k+1)\pi}{3}$.

must be $\underline{\underline{k}}$

$$x(t) = \frac{1}{3} e^{-2t} \sin 3t \sum_{n=0}^{k} \left(e^{\frac{2\pi}{3}}\right)^n$$

$$\boxed{x(t) = \frac{1}{3} e^{-2t} \sin 3t \left[\frac{e^{\frac{2\pi}{3}(k+1)} - 1}{e^{\frac{2\pi}{3}} - 1} \right].}$$

$S = 1 + z + z^2 + \dots + z^k.$

$zS = \quad z + z^2 + \dots + z^k + z^{k+1}$

$(1-z)S = 1 - z^{k+1}$

$S = \frac{1 - z^{k+1}}{1 - z} = \frac{z^{k+1} - 1}{z - 1}$

not required.

② $x'' - x' - 2x = 0 \qquad x(0) = \alpha, \; x'(0) = \beta.$

$\rightarrow x'' = 2x + x'$

$y = x'$

$y' = x'' = x' + 2x = 2x + y.$

$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{pmatrix} = 0$

$\lambda(\lambda - 1) - 2 = 0$

$\lambda^2 - \lambda - 2 = 0.$

$(\lambda - 2)(\lambda + 1) = 0$

$\lambda = 2, -1$

unstable.

Saddle.