continuation of separable eqns

Ex ~~d~~ solve

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \; ; \; y(0) = -1$$

$$\int 2(y-1)\,dy = \int 3x^2 + 4x + 2 \, dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + c$$

(general soln)

$y(0) = -1$ (everywhere I see

y, I put -1, " " "

x, I put 0)

$$(-1)^2 + 2 = 0 + 0 + 0 + c$$

$$\Rightarrow c = 3$$

then

$$y^2 - 2y = x^3 + 2x^2 + 2x + 3$$

$$y^2 - 2y - (x^3 + 2x^2 + 2x + 3) = 0$$

this has the form

$$y^2 + by + c = 0 \quad \left[\begin{array}{l} b = -2 \\ \cancel{c = -(x^3+2x^2+2x+3)} \\ c = x^3 + 2x^2 + 2x + 3 \end{array} \right]$$

use quad. form.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

therefore,

$$y_\pm = \frac{2 \pm \sqrt{4 + 4(x^3 + 2x^2 + 2x + 3}}{2}$$

$$= 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}$$

how do we choose which branch

of the square root? only one of the satisfies the I.C.

check:

$$y(0) = 1 \pm \sqrt{0+0+0+4} = 1 \pm 2$$

therefore, require negative branch $\lceil y(0) = -1 \rfloor$

so

$$y = 1 - \sqrt{x^3+2x^2+2x+4} = y_-(x)$$

- for what values of x does the solution exist?

require

$$r(x) = x^3+2x^2+2x+4 > 0$$

(require thing under $\sqrt{}$ > 0)

we find x s.t. $r(x) > 0$

notice $r(-2) = 0$ so $x+2$ is a factor x $r(x)$

synthetic division:

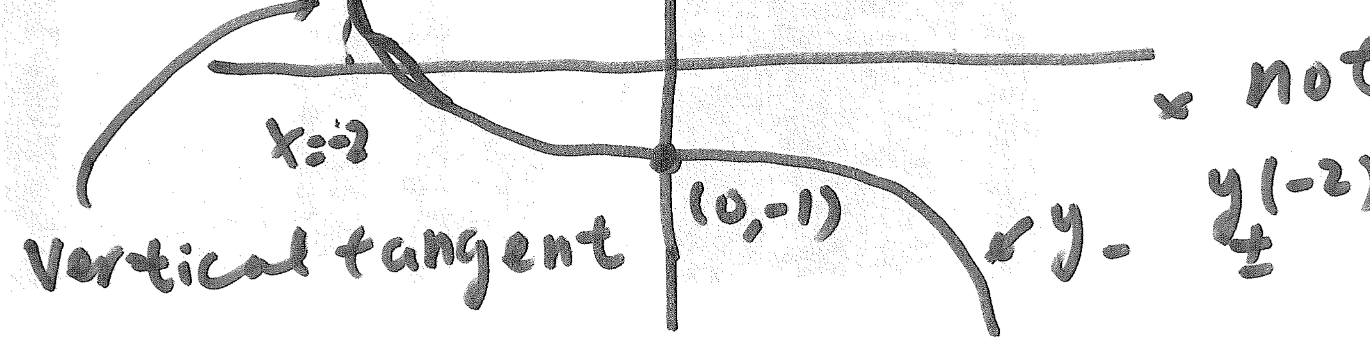
$$\begin{array}{c|cccc} 2 & 1 & 2 & 2 & 4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

so $r(x) = (x+2)(\underbrace{x^2+2})$

positive only when $x > -2$

always positive

so solution exists only when $x > -2$



note: $y_\pm(-2) = 1$

look at ODE:

$$\frac{dy}{dx} = \left(\frac{\quad}{y-1} \right. ; \text{ where}$$

$y = 1$, $|dy/dx| \to \infty$

if pose IC as $y(\underline{-2}) = 1$,
solution is not unique because
one cannot know which
branch to pick (solution curves
y_+ and y_- intersect at $(-2, 1)$)

why the non-uniqueness?
because IC posed at a point
where $f(x,y) = \dfrac{3x^2 + 4x + 2}{2(y-1)}$ is
not continuous

so existence and uniqueness not guaranteed by thm if IC posed on line $y = 1$

__Ex__ Newton's Law of cooling (solve 2 ways)

- law states that rate of change of temperature T of a body is proportional to the temperature difference with the ambient temperature T_a. Find T as a function of time t if $T(0) = T_a$.

$$\frac{dT}{dt} = K(T_a - T) \qquad k > 0,$$
$$T(0) = T_0$$

separable and linear

solve with ~~separation of variables~~

method of separable equations

$$\int \frac{dT}{T_a - T} = \int k\,dt$$

$$-\log(T_a - T) = kt + c$$

$$\log(T_a - T) = -kt + C$$

$$T_a - T = Ce^{-kt}$$

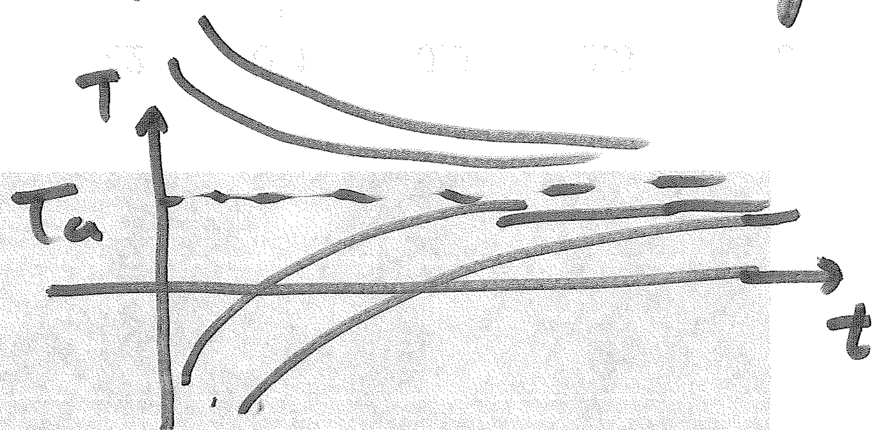
$$T = T_a - Ce^{-kt}$$

IC: $T(0) = T_a - C = T_0$

$$C = T_a - T_0$$

$$\Rightarrow \boxed{T(t) = T_a - (T_a - T_0)e^{-kt}}$$

as $t \to \infty$, $T \to T_a$ regardless of T_0



since this equation is liner, we can also solve using integrating factor (section 1.5, next time)